



**Fig. 1.** Adiabatic exponent  $\Gamma_1$ .  $\Gamma_1$  is plotted as a function of  $\lg$  internal energy [ $\text{erg g}^{-1}$ ] and  $\lg$  density [ $\text{g cm}^{-3}$ ].

**Table 1.** Opacity sources.

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XXXXXXXXXXXXXXXXXXXXXXXXXX

is a thermodynamical quantity which is of order 1 and equal to 1 for nonreacting mixtures of classical perfect gases. The physical meaning of  $\sigma_0$  and  $K$  is clearly visible in the equations above.  $\sigma_0$  represents a frequency of the order one per free-fall time.  $K$  is proportional to the ratio of the free-fall time and the cooling time. Substituting into Baker's criteria, using thermodynamic identities and definitions of thermodynamic quantities,

$$\Gamma_1 = \left( \frac{\partial \ln P}{\partial \ln \rho} \right)_S, \quad \chi_\rho = \left( \frac{\partial \ln P}{\partial \ln \rho} \right)_T, \quad \kappa_P = \left( \frac{\partial \ln \kappa}{\partial \ln P} \right)_T$$

$$\nabla_{\text{ad}} = \left( \frac{\partial \ln T}{\partial \ln P} \right)_S, \quad \chi_T = \left( \frac{\partial \ln P}{\partial \ln T} \right)_\rho, \quad \kappa_T = \left( \frac{\partial \ln \kappa}{\partial \ln T} \right)_T$$

one obtains, after some pages of algebra, the conditions for *stability* given below:

$$\frac{\pi^2}{8} \frac{1}{\tau_{\text{ff}}^2} (3\Gamma_1 - 4) > 0 \quad (6)$$

$$\frac{\pi^2}{\tau_{\text{co}} \tau_{\text{ff}}^2} \Gamma_1 \nabla_{\text{ad}} \left[ \frac{1 - 3/4 \chi_\rho}{\chi_T} (\kappa_T - 4) + \kappa_P + 1 \right] > 0 \quad (7)$$

$$\frac{\pi^2}{4} \frac{3}{\tau_{\text{co}} \tau_{\text{ff}}^2} \Gamma_1^2 \nabla_{\text{ad}} \left[ 4\nabla_{\text{ad}} - (\nabla_{\text{ad}} \kappa_T + \kappa_P) - \frac{4}{3\Gamma_1} \right] > 0 \quad (8)$$

For a physical discussion of the stability criteria see Baker ((1966)) or Cox ((1980)).

We observe that these criteria for dynamical, secular and vibrational stability, respectively, can be factorized into

1. a factor containing local timescales only,
2. a factor containing only constitutive relations and their derivatives.

The first factors, depending on only timescales, are positive by definition. The signs of the left hand sides of the inequalities (6), (7) and (8) therefore depend exclusively on the second factors containing the constitutive relations. Since they depend only on state variables, the stability criteria themselves are *functions of the thermodynamic state in the local zone*. The one-zone stability can therefore be determined from a simple equation of state, given for example, as a function of density and temperature. Once the microphysics, i.e. the thermodynamics and opacities (see Table ??), are specified (in practice by specifying a chemical composition) the one-zone stability can be inferred if the thermodynamic state is specified. The zone – or in other words the layer – will be stable or unstable in whatever object it is imbedded as long as it satisfies the one-zone-model assumptions. Only the specific growth rates (depending upon the time scales) will be different for layers in different objects.

We will now write down the sign (and therefore stability) determining parts of the left-hand sides of the inequalities (6), (7) and (8) and thereby obtain *stability equations of state*.

The sign determining part of inequality (6) is  $3\Gamma_1 - 4$  and it reduces to the criterion for dynamical stability

$$\Gamma_1 > \frac{4}{3}. \quad (9)$$

**Fig. 2.** Vibrational stability equation of state  $S_{\text{vib}}(\lg e, \lg \rho)$ .  $> 0$  means vibrational stability.

Stability of the thermodynamical equilibrium demands

$$\chi_\rho > 0, \quad c_v > 0, \quad (10)$$

and

$$\chi_T > 0 \quad (11)$$

holds for a wide range of physical situations. With

$$\Gamma_3 - 1 = \frac{P}{\rho T} \frac{\chi_T}{c_v} > 0 \quad (12)$$

$$\Gamma_1 = \chi_\rho + \chi_T (\Gamma_3 - 1) > 0 \quad (13)$$

$$\nabla_{\text{ad}} = \frac{\Gamma_3 - 1}{\Gamma_1} > 0 \quad (14)$$

we find the sign determining terms in inequalities (7) and (8) respectively and obtain the following form of the criteria for dynamical, secular and vibrational *stability*, respectively:

$$3\Gamma_1 - 4 =: S_{\text{dyn}} > 0 \quad (15)$$

$$\frac{1 - 3/4 \chi_\rho}{\chi_T} (\kappa_T - 4) + \kappa_P + 1 =: S_{\text{sec}} > 0 \quad (16)$$

$$\text{XXXXXXXXXXXXXXXX} = X \quad (17)$$

The constitutive relations are to be evaluated for the unperturbed thermodynamic state (say  $(\rho_0, T_0)$ ) of the zone. We see that the one-zone stability of the layer depends only on the constitutive relations  $\Gamma_1$ , XXXXXXXXXXXX XXXXXXXXXXXX XXXXXXXXXXXX. These depend only on the unperturbed thermodynamical state of the layer. Therefore the above relations define the one-zone-stability equations of state  $S_{\text{dyn}}$ ,  $S_{\text{sec}}$  and  $S_{\text{vib}}$ . See Fig. 2 for a picture of  $S_{\text{vib}}$ . Regions of secular instability are listed in Table 1.

### 3. Conclusions

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The conditions for the stability of static, radiative layers in gas spheres, as described by Baker's ((1966)) standard one-zone model, can be expressed as stability equations of state. These stability equations of state depend only on the local thermodynamic state of the layer.

If the constitutive relations – equations of state and Rosseland mean opacities – are specified, the stability equations of state can be evaluated without specifying properties of the layer.

For solar composition gas the  $\kappa$ -mechanism is working in the regions of the ice and dust features in the opacities, the  $\text{H}_2$  dissociation and the combined H, first He ionization zone, as indicated by vibrational instability. These regions of instability are much larger in extent and degree of instability than the second He ionization zone that drives the Cepheid pulsations.

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