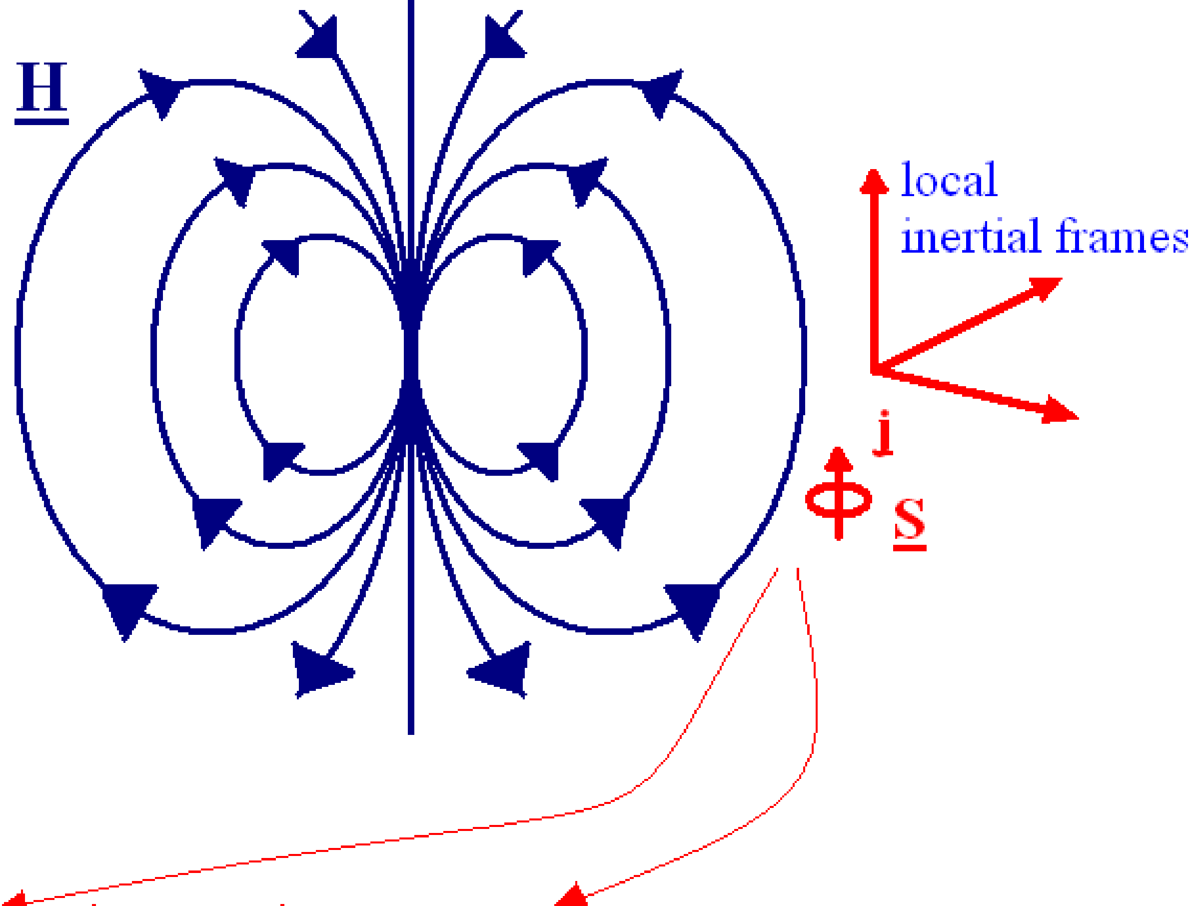
The LARES and LAGEOS Satellites and the Accurate Measurement of Gravitomagnetism

Ignazio Ciufolini, Univ. Salento and INFN
Antonio Paolozzi, Sapienza Univ. Roma and INFN
Erricos Pavlis, UMBC

THIS TALK IS DEDICATED TO JOHN ARCHIBALD WHEELER ONE OF THE MASTERS OF PHYSICS OF XX CENTURY AND ONE OF THE FATHERS OF THE REINASSANCE OF GENERAL RELATIVITY



$$\mathbf{F} = \left(\frac{1}{2}\mathbf{S}\cdot\nabla\right)\mathbf{H} \qquad \tau \cong \tfrac{1}{2}\mathbf{S}\times\mathbf{H} = \frac{d\mathbf{S}}{dt} \equiv \dot{\mathbf{\Omega}}\times\mathbf{S}$$

$$\dot{\mathbf{\Omega}} = -\tfrac{1}{2}\mathbf{H} = \frac{-\mathbf{J} + 3\,(\mathbf{J}\cdot\hat{\mathbf{x}})\,\hat{\mathbf{x}}}{|\mathbf{x}|^3}$$

Dragging of inertial frames:

Mach principle in general relativity

GRAVITATION AND INERTIA

I.C. and J.A. Wheleer -1995

INVARIANT CHARACTERIZATION of GRAVITOMAGNETISM

By explicit spacetime invariants built with the Riemann tensor:

I.C. 1994

I.C. and Wheeler 1995:

for the Kerr metric:

$\frac{1}{2}\,\epsilon_{\alpha\beta\sigma\rho}\,R^{\sigma\rho}{}_{\mu\nu}\,R^{\alpha\beta\mu\nu} = 1536\,J\,M\cos\theta\,(\rho^5\rho^{-6} - \rho^3\rho^{-5} + 3/16\,\rho\,\rho^{-4})$

In weak-field and slow-motion:

$^{*}R \cdot R = 288\,(J\,M)/r^7 \cos\theta + \cdots$

J = aM = angular momentum

This gravitomagnetic invariant is null on the ecliptic plane and substantially null on the Moon orbit: I.C. arXiv:0809.3219v1 [gr-qc] 18 Sep 2008

SOME EXPERIMENTAL ATTEMPTS TO MEASURE FRAME-DRAGGING AND GRAVITOMAGNETISM

- **1896: Benedict and Immanuel FRIEDLANDER**
  (torsion balance near a heavy flying-wheel)
- **1904: August FOPPL (Earth-rotation effect on a gyroscope)**
- **1916: DE SITTER (shift of perihelion of Mercury due to Sun rotation)**
- **1918: LENSE AND THIRRING (perturbations of the Moons of solar system planets by the planet angular momentum)**
- **1959: Yilmaz (satellites in polar orbit)**
- **1976: Van Patten-Everitt**
  (two non-passive counter-rotating satellites in polar orbit: a very expensive experiment)
- **1960: Schiff-Fairbank-Everitt (Earth orbiting gyroscopes)**
- **1977-78: Cugusi and Proverbio, on LAGEOS only (however, wrong rate for frame-dragging)**
- **1986: I.C.: USE THE NODES OF TWO LAGEOS SATELLITES**
  (two supplementary inclination, passive, laser ranged satellites)
- **1988 : Nordtvedt (Astrophysical evidence from periastron**
  rate of binary pulsar)
- **1995-2007: I.C. et al. (obs. & measurements using LAGEOS and LAGEOS-II)**
- **1998: Some astrophysical evidence from accretion disks of black holes and neutron stars, LLR observations**
- **2004 launch of Gravity Probe B**
- **2009 LARES**

GRAVITY PROBE B

Some serious problems with the GP-B data analysis have been recently outlined, see, for example Prof. O'Connel :
http://www.phys.lsu.edu/faculty/oconnell/oconnell_pubs.html
(pub. number 307)
R. F. O'Connell, "Gravito-Magnetism in one-body and two-body systems: Theory and Experiment", in, "Atom Optics and Space Physics", Proc. of Course CLXVIII of the International School of Physics "Enrico Fermi", Varenna, Italy, 2007, ed. E. Arimondo, W. Ertmer and W. Schleich, to be published; and
G. Forst 2008: http://arxiv.org/PS_cache/arxiv/pdf/0712/0712.3934v1.pdf

R: A WORK IN PROGRESS

eiser, and J. Turneaure

ntifying the Error

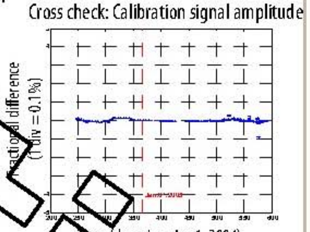
data for one parameter
nalysis with degraded data
lativity result to original
periment error for parameter

ency
confidence in result
results decreases error

build confidence
reference parameters

ganize effort

Cross check: Calibration signal amplitude



l relativity uncertainty associated with the modeling of
ne systematic error of that effect.

pe, Roll Phase, Timing

Gyro Torques (I)

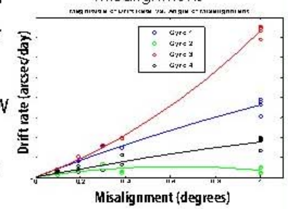
Misalignment torque: Torque proportional to angle subtended by SV roll axis and gyro spin axis

Cause: Interaction of patch fields on rotor and casing

Induced drift rate ~0.1 to 1.0 arcsec/yr

Require > 99 % removal for 10^{-3} arcsec/yr

Measured drift rate vs. misalignment

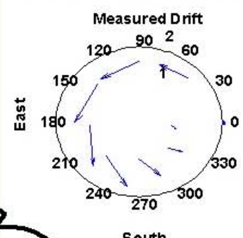


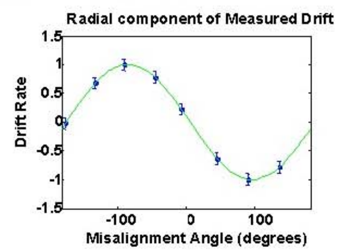
Misalignment torque: direction is know

Mitigation: phase sensitive detection
measure relativity in orthogonal direction

Error in resulting mitigated drift:

Uncertainty in torque direction: gives 4 marcsec/yr
Changing amplitudes of misalignment, torque coefficient





Next Steps

Current Limits on Experiment Error

Error Source	Current Error (marcsec/yr)	Mitigation
Current error including resonance observation	100	Improved modeling
EMI effects	5	Data grading & noise removal
Misalignment patch-effect torque	4 *	Improved misalign-ment model
Polhode frequency error (Cg)	2.5	Trapped flux mapping.
Roll phase		None

Measurement of the Lense-Thirring Drag on High-Altitude, Laser-Ranged Artificial Satellites

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(Received 16 October 1984; revised manuscript received 19 April 1985)

We describe a new method of measuring the Lense-Thirring relativistic nodal drag using LAGEOS together with another similar high-altitude, laser-ranged satellite with appropriately chosen orbital parameters. We propose, for this purpose, that a future satellite such as LAGEOS II have an inclination supplementary to that of LAGEOS. The experiment proposed here would provide a method for experimental verification of the general relativistic formulation of Mach's principle and measurement of the gravitomagnetic field.

PACS numbers: 04.80.+z

In special and general relativity there are several precession phenomena associated with the angular momentum vector of a body. If a test particle is orbiting a rotating central body, the plane of the orbit of the particle is dragged by the intrinsic angular momentum J of the central body, in agreement with the general relativistic formulation of Mach's principle.[1]

In the weak-field and slow-motion limit the nodal lines are dragged in the sense of rotation, at a rate given by[2]

$$\dot{\Omega} = [2/a^3(1-e^2)^{3/2}]J, \qquad (1)$$

where a is the semimajor axis of the orbit, e is the eccentricity of the orbit, and geometrized units are used, i.e., $G=c=1$. This phenomenon is the Lense-Thirring effect, from the names of its discoverers in 1918.[2]

In addition to this there are other precession phenomena associated with the intrinsic angular momentum or spin $\mathbf{S}$ of an orbiting particle. In the weak-field and slow-motion limit the vector $\mathbf{S}$ precesses at a rate given by[1] $d\mathbf{S}/d\tau = \dot{\mathbf{\Omega}} \times \mathbf{S}$ where

$$\dot{\mathbf{\Omega}} \equiv -\tfrac{1}{2}\mathbf{v}\times\mathbf{a} + \tfrac{3}{2}\mathbf{v}\times\nabla U + \frac{1}{r^3}\left[-\mathbf{J} + \frac{3(\mathbf{J}\cdot\mathbf{r})\mathbf{r}}{r^2}\right], \qquad (2)$$

where $\mathbf{v}$ is the particle velocity, $\mathbf{a} \equiv d\mathbf{v}/d\tau - \nabla U$ is its nongravitational acceleration, $\mathbf{r}$ is its position vector, τ is its proper time, and U is the Newtonian potential.

The first term of this equation is the Thomas precession.[3] It is a special relativistic effect due to the noncommutativity of nonaligned Lorentz transformations. It may also be viewed as a coupling between the particle velocity $\mathbf{v}$ and the nongravitational forces acting on it.

The second (de Sitter[4]–Fokker[5]) term is general relativistic, arising even for a nonrotating source, from the parallel transport of a direction defined by $\mathbf{S}$; it may be viewed as spin precession due to the coupling between the particle velocity $\mathbf{v}$ and the static —$g_{\alpha\beta,0}=0$ and $g_{i0}=0$—part of the space-time geometry.

The third (Schiff[6]) term gives the general relativistic precession of the particle spin $\mathbf{S}$ caused by the intrinsic angular momentum $\mathbf{J}$ of the central body—$g_{i0}\neq 0$.

We also mention the precession of the periapsis of an orbiting test particle due to the angular momentum of the central body. This tiny shift of the perihelion of Mercury due to the rotation of the Sun was calculated by de Sitter in 1916.[7]

All these effects are quite small for an artificial satellite orbiting the Earth.

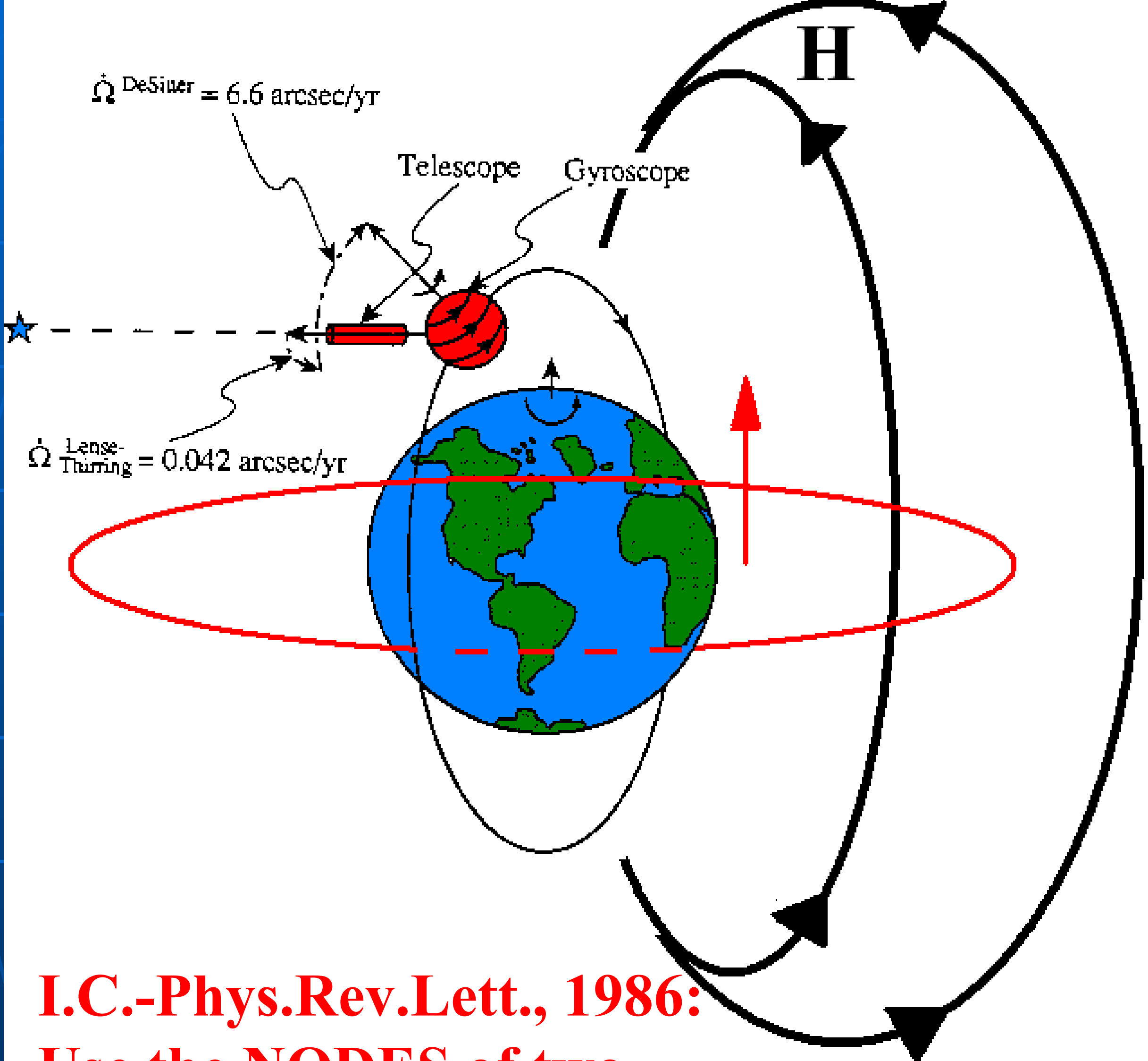
We propose here to measure the Lense-Thirring dragging by measuring the nodal precession of laser-ranged Earth satellites. We shall show that two satellites would be required; we propose that LAGEOS[8–10] together with a second satellite LAGEOS X with opposite inclination (i.e., with $I^X = 180° - I$, where $I \simeq 109.94°$ is the orbital inclination of LAGEOS) would provide the needed accuracy.

The major part of the nodal precession of an Earth satellite is a classical effect due to deviations from spherical symmetry of the Earth's gravity field —quadrupole and higher mass moments.[11] These deviations from sphericity are measured by the expansion of the potential $U(r)$ in spherical harmonics. From this expansion of $U(r)$ follows[11] the formula for the classical precession of the nodal lines of an Earth satellite:

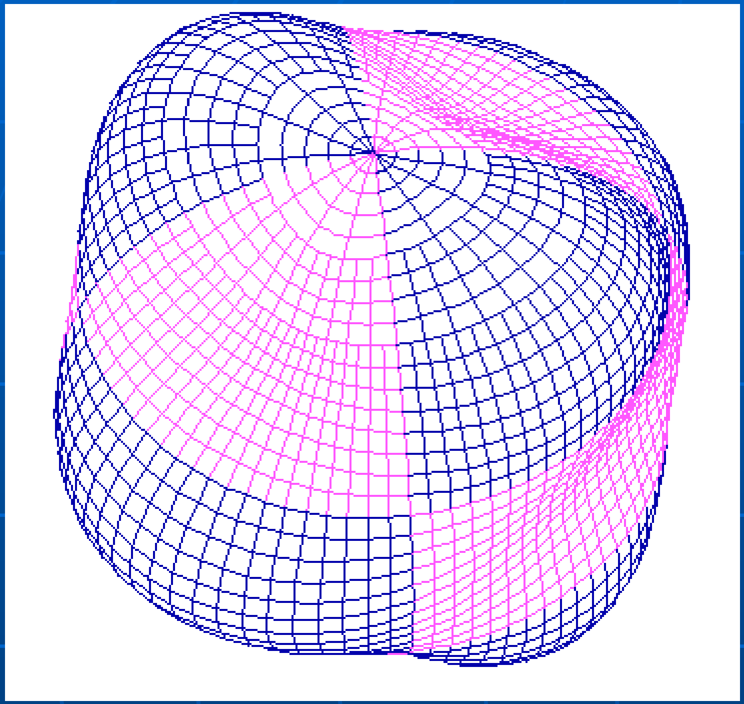
$$\dot{\Omega}_{\text{class}} \simeq -\frac{3}{2}n\left(\frac{R_\oplus}{a}\right)^2 \frac{\cos I}{(1-e^2)^2}\left\{J_2 + J_4\left[\frac{5}{8}\left(\frac{R_\oplus}{a}\right)^2(7\sin^2 I - 4)\frac{1+\frac{3}{2}e^2}{(1-e^2)^2}\right] + \dots\right\}, \qquad (3)$$

IC, PRL 1986:
Use of the nodes of two laser-ranged satellites to measure the Lense-Thirring effect

LAGEOS III:
in 1989 proposed by CSR-UT, CfR-UT and CNR-Rome to NASA and ASI and
LARES:
proposed to ASI in 1998 by Univ. of Rome and CNR

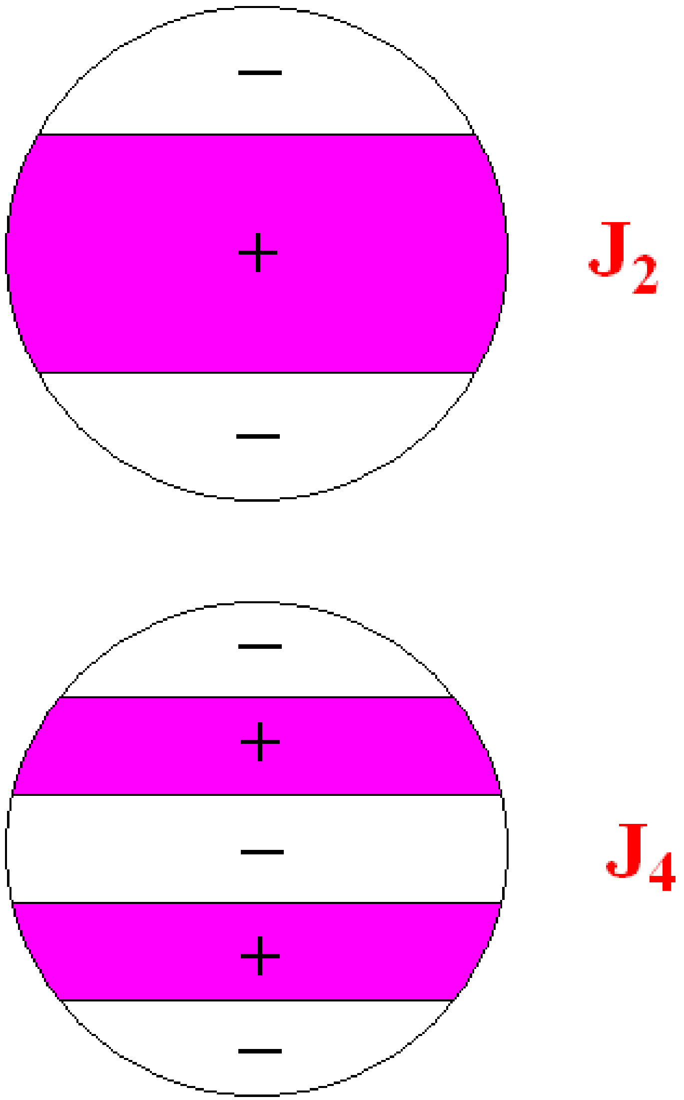


I.C.-Phys.Rev.Lett., 1986: Use the NODES of two LAGEOS satellites.



l=3, m=1

EVEN ZONAL HARMONICS



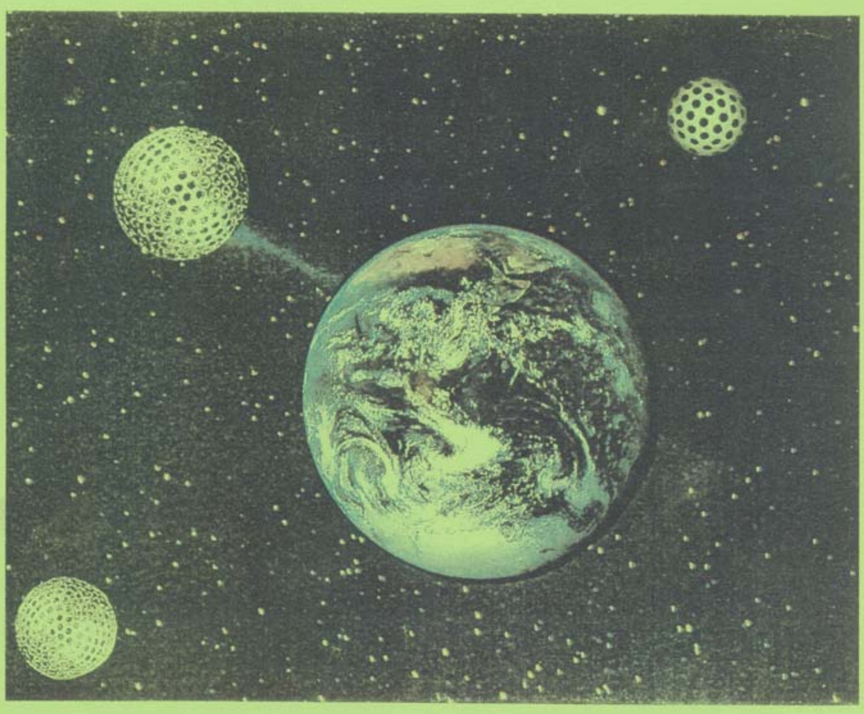
LARES

WEBER-SAT

A NEW SATELLITE FOR THE LARES EXPERIMENT

LAser RElativity experimentS

for
Testing General Relativity and Studying the Earth Gravitational Field



January 2003

MAIN COLLABORATION

I.C.–LARES Principal Investigator

- **University of Salento and INFN**

A. Paolozzi

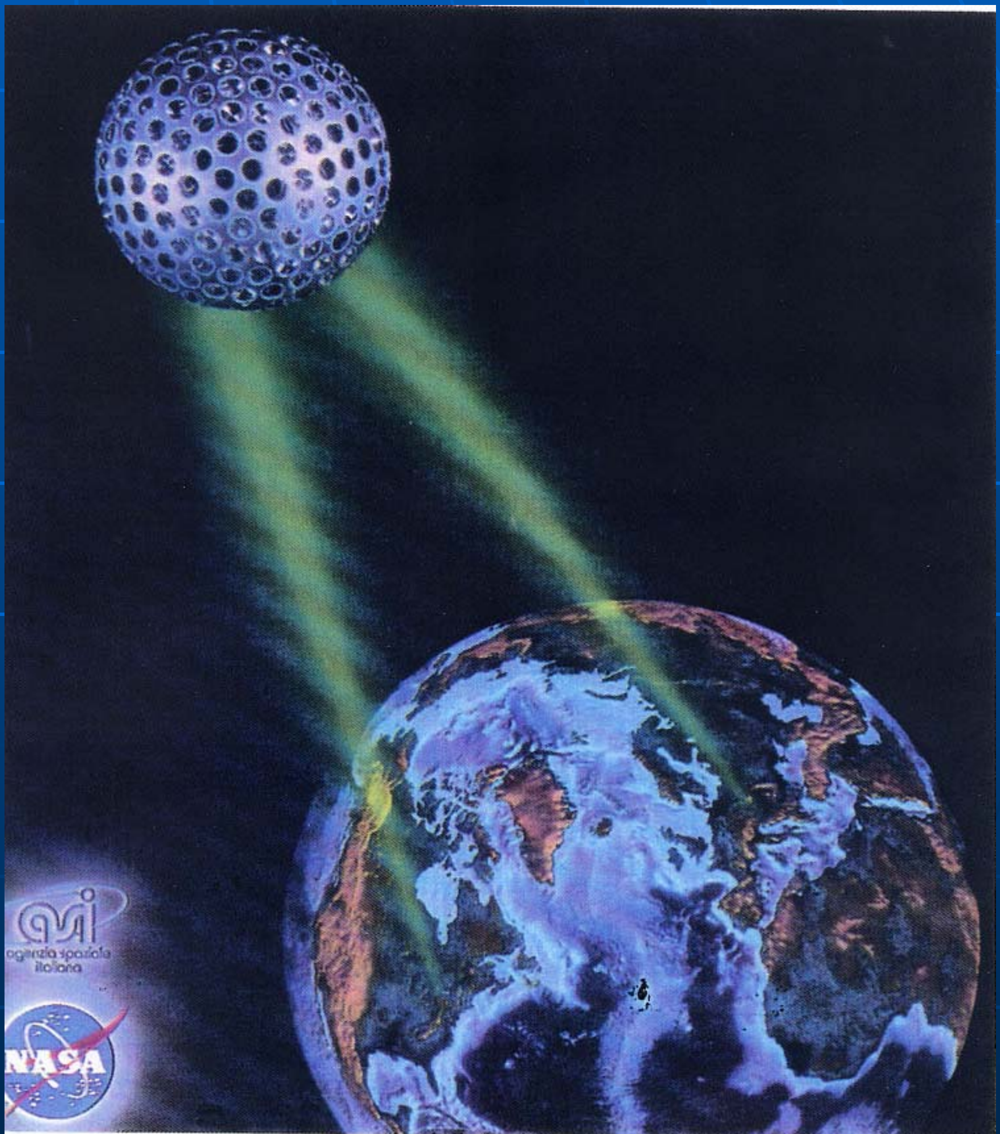
- **“Sapienza” University of Roma and INFN**
- **LNF-INFN**

E. Pavlis

- **University of Maryland BC**
- **NASA-Goddard**
- **University of Texas at Austin**
- **GFZ-Potsdam/Munich**
- **AstroSpace Center of Lebedev Phys. Inst.-Moscow**

Lageos II: 1992

However, NO LAGEOS satellite with supplementary inclination to LAGEOS has ever been launched. Nevertheless, LAGEOS II was launched in 1992.



International Journal of Modern Physics A, Vol. 4, No. 13 (1989) 3083–3145

A COMPREHENSIVE INTRODUCTION TO THE LAGEOS GRAVITOMAGNETIC EXPERIMENT: FROM THE IMPORTANCE OF THE GRAVITOMAGNETIC FIELD IN PHYSICS TO PRELIMINARY ERROR ANALYSIS AND ERROR BUDGET

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CNR-Istituto di Fisica dello Spazio Interplanetario, Via G. Galilei-CP 27-00044 Frascati, Italy

and

Center for Space Research, The University of Texas at Austin, Austin, Texas 78712, USA

Received 3 May 1988
Revised 7 October 1988

The existence of the gravitomagnetic field, generated by mass currents according to Einstein geometrodynamics, has never been proved. The author of this paper, after a discussion of the importance of the gravitomagnetic field in physics, describes the experiment that he proposed in 1984 to measure this field using LAGEOS (Laser geodynamics satellite) together with another non-polar, laser-ranged satellite with the same orbital parameters as LAGEOS but a supplementary inclination.

The author then studies the main perturbations and measurement uncertainties that may affect the measurement of the Lense-Thirring drag. He concludes that, over the period of the node of ~ 3 years, the maximum error, using two nonpolar laser ranged satellites with supplementary inclinations, should not be larger than ~ 10% of the gravitomagnetic effect to be measured.

Table of Contents

IC IJMPA 1989: Analysis of the orbital perturbations affecting the nodes of LAGEOS-type satellites

(1) Use two LAGEOS satellites with supplementary inclinations

OR:

Use n satellites of LAGEOS-type to measure the first n-1 even zonal harmonics: J_2, J_4, … and the Lense-Thirring effect

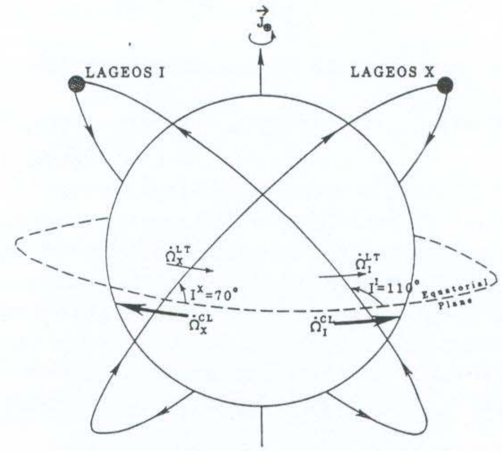


Fig. 5. The LAGEOS and LAGEOS *X* orbits and their classical and gravitomagnetic nodal precessions. A new[17] configuration to measure the Lense-Thirring effect.

For J_2, this corresponds, from formula (3.2), to an uncertainty in the nodal precession of 450 milliarcsec/year, and similarly for higher J_{2n} coefficients. Therefore, the uncertainty in $\dot{\Omega}^{\text{Class}}_{\text{Lageos}}$ is more than ten times larger than the Lense-Thirring precession.

A solution would be to orbit several high-altitude, laser-ranged satellites, similar to LAGEOS, to measure J_2, J_4, J_6, etc., and one satellite to measure $\dot{\Omega}^{\text{Lense-Thirring}}$.

Another solution would be to orbit polar satellites; in fact, from formula (3.2), for polar satellites, since $I = 90°$, $\dot{\Omega}^{\text{Class}}$ is equal to zero. As mentioned before, Yilmaz proposed the use of polar satellites in 1959.[40,41] In 1976, Van Patten and Everitt[46,47] proposed an experiment with two drag-free, guided, counter-rotating, polar satellites to avoid inclination measurement errors.

A new solution[15,16,17,21,22,23] would be to orbit a second satellite, of LAGEOS type, with the same semimajor axis, the same eccentricity, but the inclination supplementary to that of LAGEOS (see Fig. 5). Therefore, "LAGEOS X" should have the following orbital parameters:

$$I^X \cong \pi - I^I \cong 70°\,, \qquad a^X \cong a^I\,, \qquad e^X \cong e^I\,. \tag{3.3}$$

With this choice, since the classical precession $\dot{\Omega}^{\text{Class}}$ is linearly proportional to $\cos I$, $\dot{\Omega}^{\text{Class}}$ would be equal and opposite for the two satellites:

$$\dot{\Omega}^{\text{Class}}_X = -\dot{\Omega}^{\text{Class}}_I\,. \tag{3.4}$$

By contrast, since the Lense-Thirring precession $\dot{\Omega}^{\text{Lense-Thirring}}$ is independent of the inclination (Eq. (3.1)), $\dot{\Omega}^{\text{Lense-Thirring}}$ will be the same in magnitude and sign for both satellites:

IC
Nuovo Cimento A
1996

for LAGEOS II: $\dot{\omega}_{\mathrm{LAGEOS\,II}} \cong 160°/\mathrm{year}$, and the classical perigee precession is:

$$(11) \quad \dot{\omega}^{\mathrm{Class}} = -\frac{3}{4}\, n \left(\frac{R_\oplus}{a}\right)^2 \frac{1 - 5\cos^2 I}{(1-e^2)^2}\, J_2 - \\ -[[15 n R_\oplus^4 (108 + 135 e^2 + 208\cos(2I) + 252 e^2 \cos(2I) + 196\cos(4I) + \\ + 189 e^2 \cos(4I))]/(1024 a^4 (1-e^2)^4)]\, J_4 + \Sigma P_{2n} \times J_{2n}\,,$$

where the P_{2n} are the coefficients (in the equation for the perigee rate) of the nonnormalized even zonal harmonics $J_{2n} \equiv -\sqrt{4n+1}\, C_{2n0}$. Thus, for the perigee of LAGEOS II, one has (in units of $\dot{\omega}_{\mathrm{II}}^{\text{Lense-Thirring}}$):

	$\delta\dot{\omega}_{\mathrm{II}}/\dot{\omega}_{\mathrm{II}}^{\mathrm{L\text{-}T}}$ due to JGM3 estimated errors	$\delta\dot{\omega}_{\mathrm{II}}/\dot{\omega}_{\mathrm{II}}^{\mathrm{L\text{-}T}}$ due to difference (JGM3 − GEMT3)
δC_{20}	~ 1.1	~ 5.9
δC_{40}	~ 2.1	~ 5.3
δC_{60}	~ 0.41	~ 0.32
δC_{80}	~ 0.68	~ 0.8
$\delta C_{10,0}$	~ 0.22	~ 0.07

From these uncertainties in the perigee rate of LAGEOS II, similarly to what inferred for the nodal rates, it is manifest that the dominating error sources are due to the uncertainties in C_{20} and C_{40}.

Thus, summarizing, we have now the three unknowns δC_{20}, δC_{40} and Lense-Thirring effect, and the three observable quantities $\dot{\Omega}_{\mathrm{LAGEOS}}$, $\dot{\Omega}_{\mathrm{LAGEOS\,II}}$, and $\dot{\omega}_{\mathrm{LAGEOS\,II}}$.

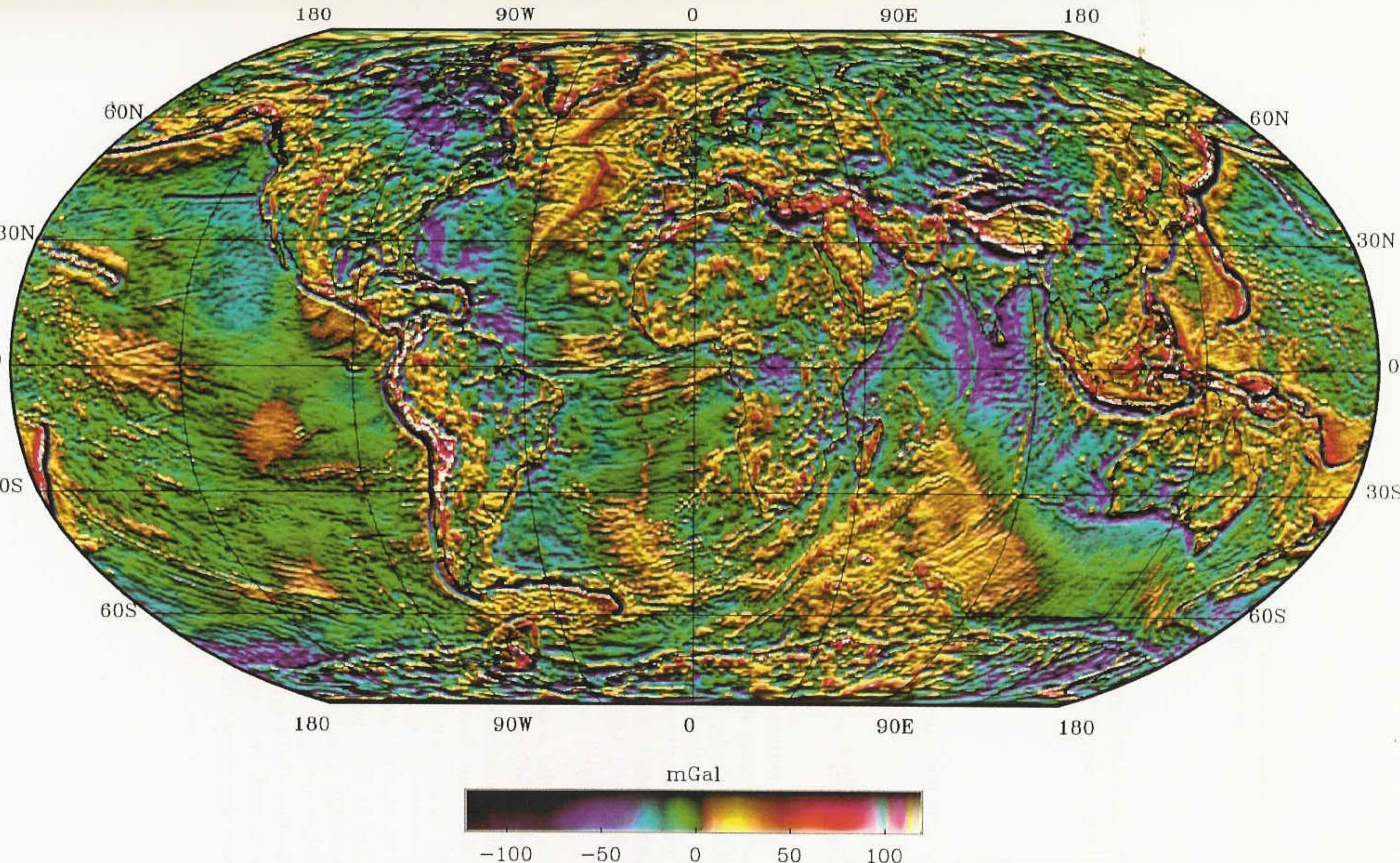
The main unmodeled part of the LAGEOS I nodal rate, due to the uncertainties in the even zonal harmonics, to the errors in the value of the orbital parameters (mainly the inclination), and including the Lense-Thirring effect (to be determined), is:

$$(12) \quad \delta\dot{\Omega}_I = (-9.3\cdot 10^{11}) \times \delta C_{20} - (4.62\cdot 10^{11}) \times \delta C_{40} + \Sigma N_{2n} \times \delta C_{2n0} + 6 \times \delta I_I + 31\mu\,,$$

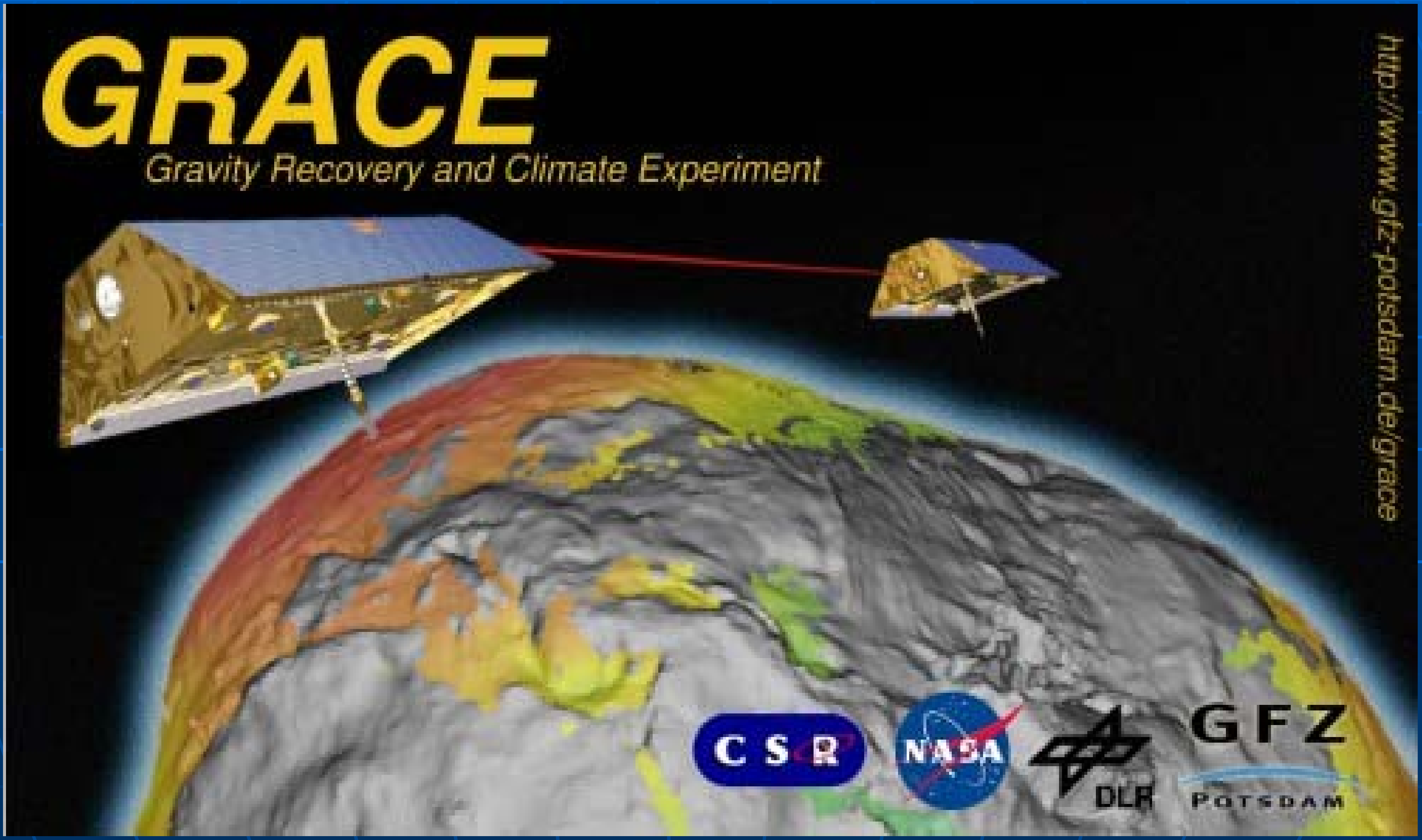
where $\delta\dot{\Omega}$ is in units of milliarcsec/year, and δI in milliarcsec. This formula shows the main error sources in the calculated nodal rate (apart from the errors due to tides and to nongravitational perturbations; see below). In this formula the first two contributions are due to the uncertainties δC_{20} and δC_{40}, we then have the error due to the uncertainties in the higher even zonal harmonics δC_{2n0} (with $2n \geqslant 6$), and the error due to the uncertainties in the determination of the inclination δI_I. In this formula we have also included the Lense-Thirring [2] parameter μ, by definition 1 in general relativity: $\mu^{\mathrm{GR}} \equiv 1$, that, if not incorporated in the modeling of the orbital perturbations, will affect the orbital residuals. One can write a similar expression for the node of LAGEOS II:

$$(13) \quad \delta\dot{\Omega}_{\mathrm{II}} = (17.17\cdot 10^{11}) \times \delta C_{20} + \\ + (1.68\cdot 10^{11}) \times \delta C_{40} + \Sigma N''_{2n} \times \delta C_{2n0} + 5.3 \times \delta I_{\mathrm{II}} + 31.5\mu$$

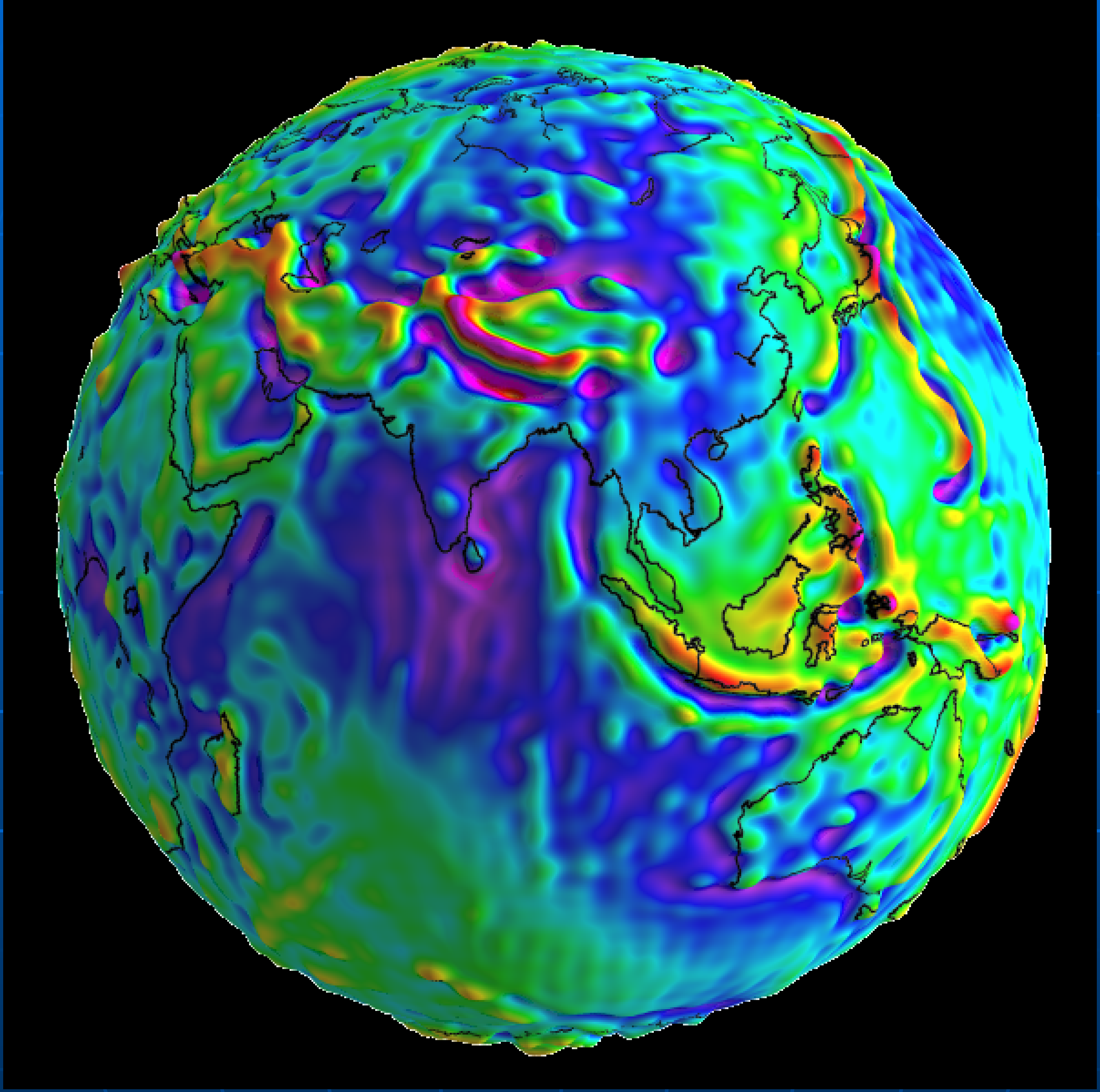
1996 observations used the **EGM-96 GRAVITY MODEL**



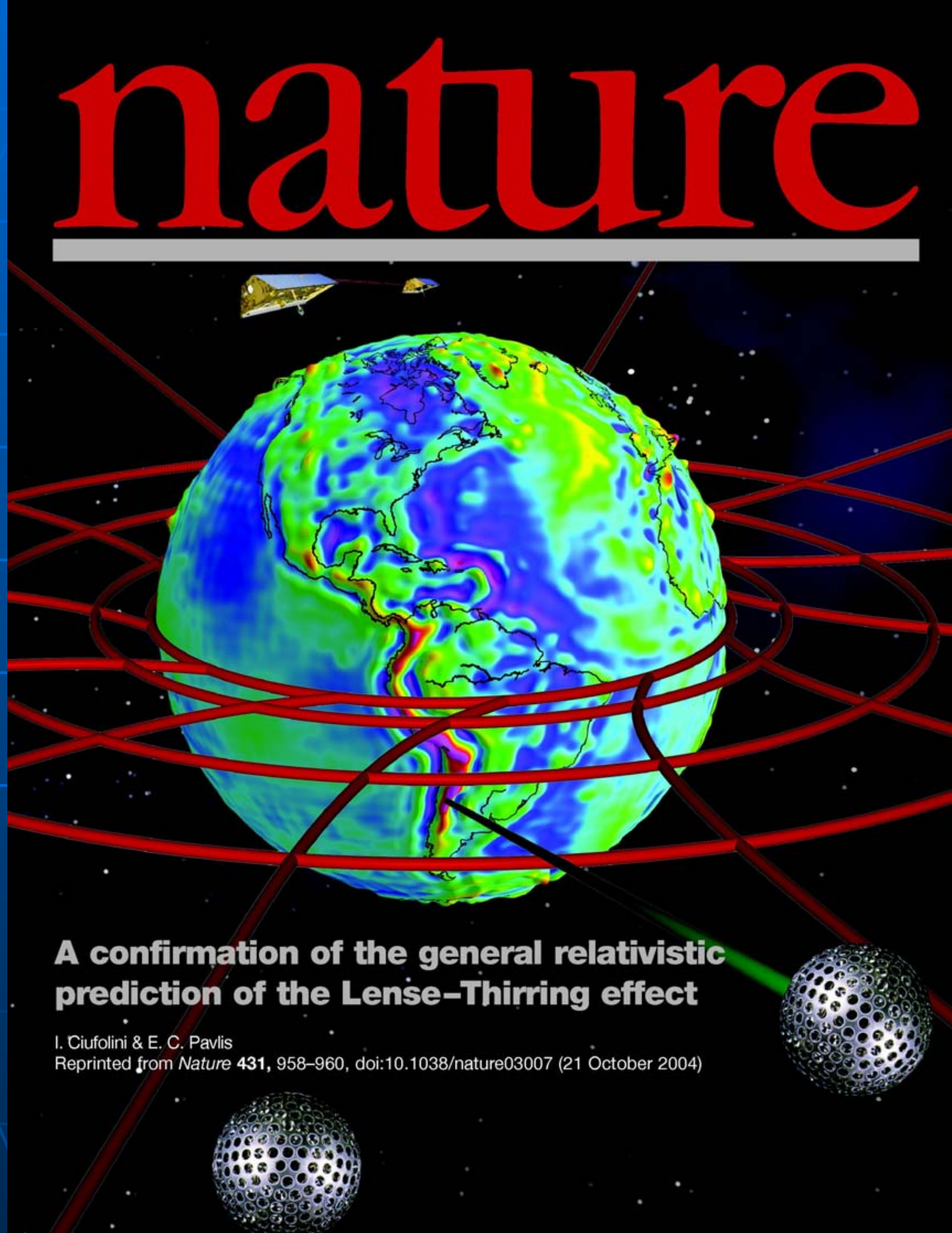
2002



Use of GRACE to test Lense-Thirring at a few percent level:
J. Ries et al. 2003 (1999),E. Pavlis 2002 (2000)



EIGEN-GRACE-S (GFZ 2004)


nature
A confirmation of the general relativistic prediction of the Lense–Thirring effect
I. Ciufolini & E. C. Pavlis
Reprinted from Nature 431, 958–960, doi:10.1038/nature03007 (21 October 2004)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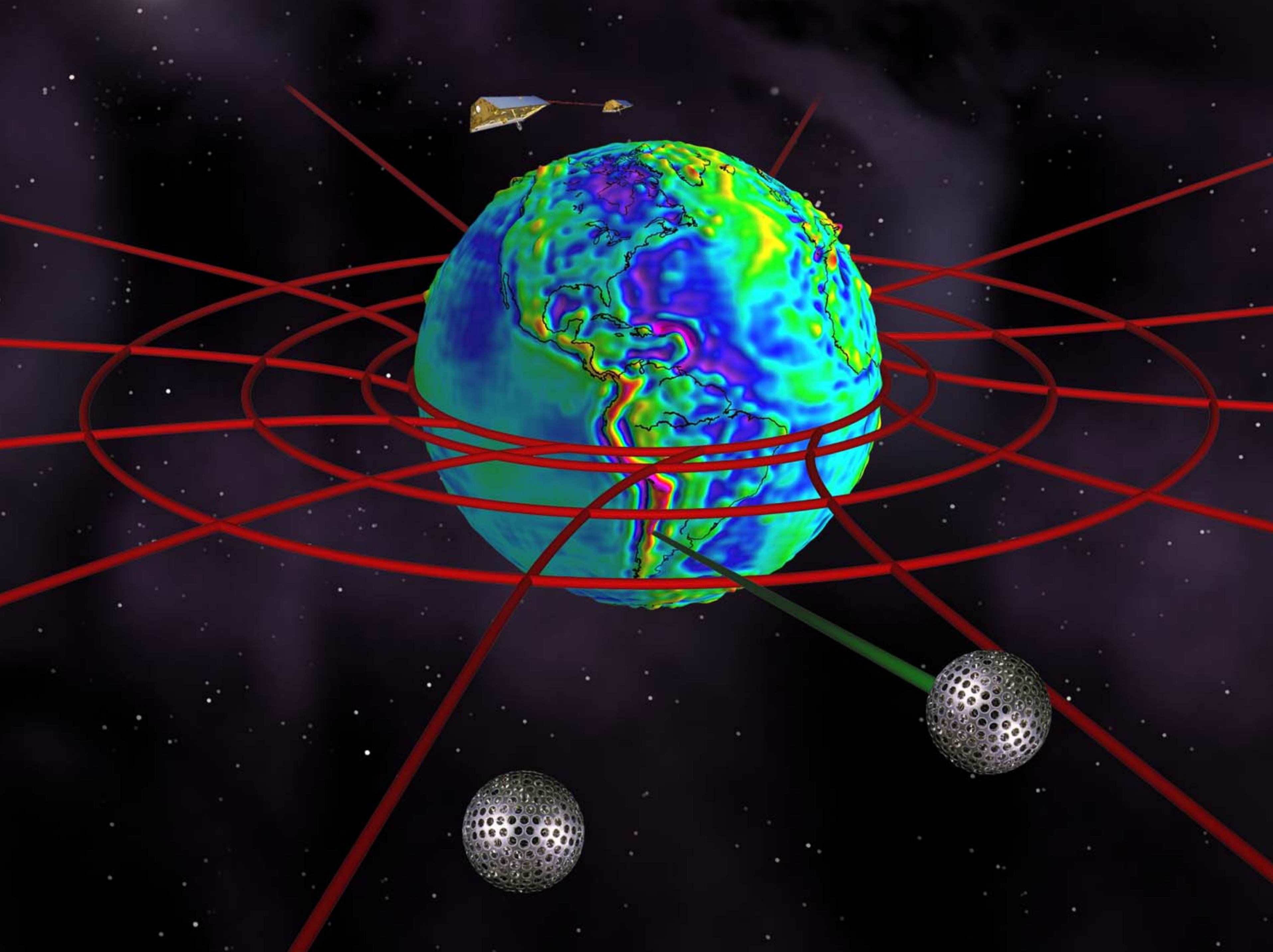


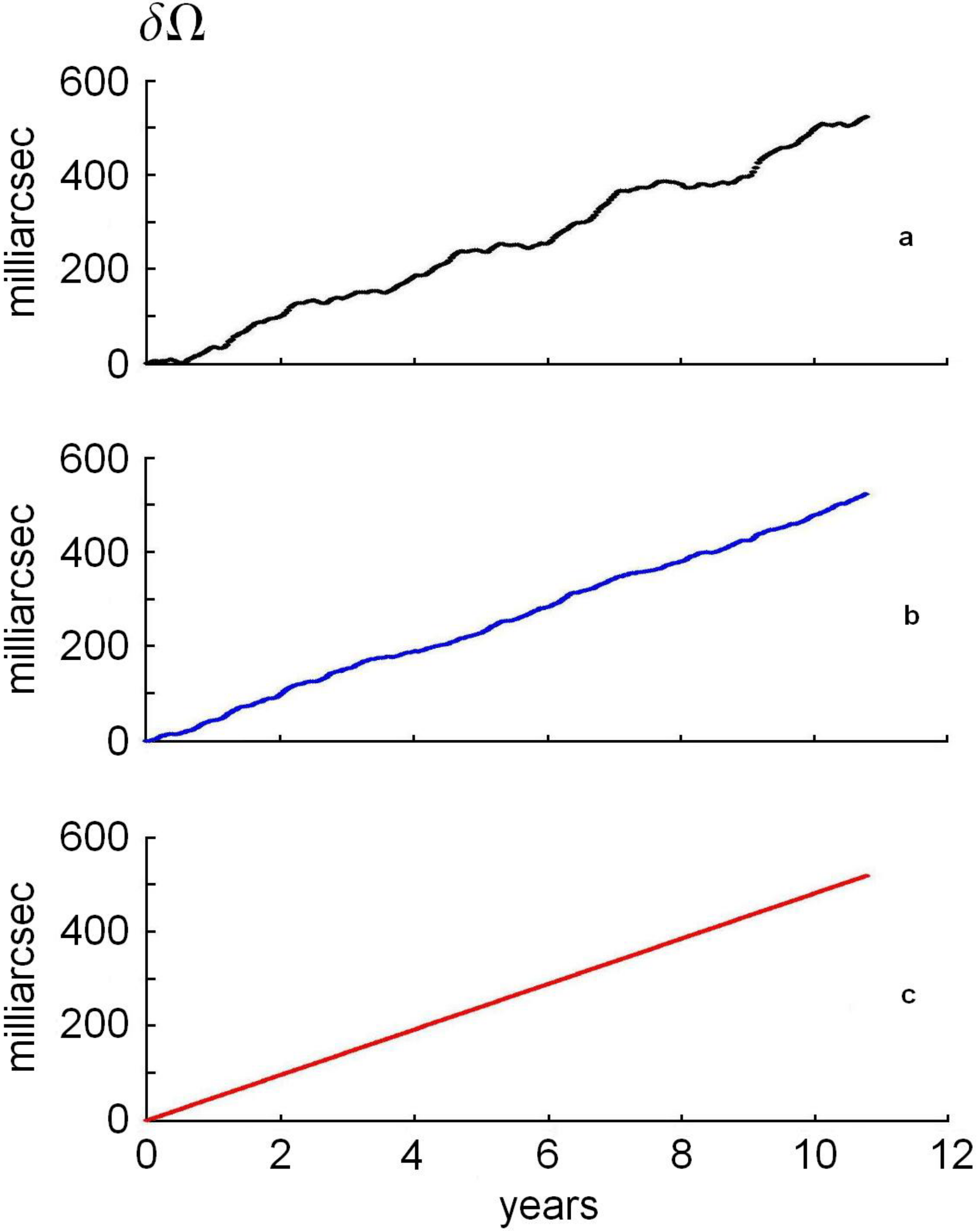


Figure 2

Observed value of Lense-Thirring effect using The combination of the LAGEOS nodes.

Observed value of Lense-Thirring effect = 99% of the general relativistic prediction. Fit of linear trend plus 6 known frequencies

General relativistic Prediction = 48.2 mas/yr

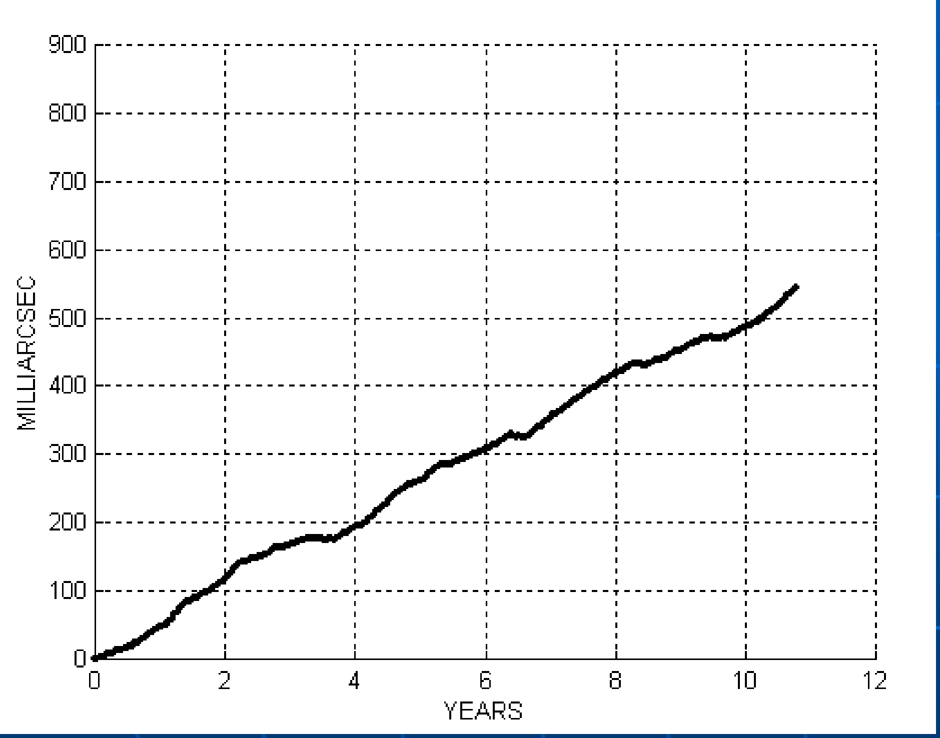
I.C. & E.Pavlis, Letters to NATURE, 431, 958, 2004.

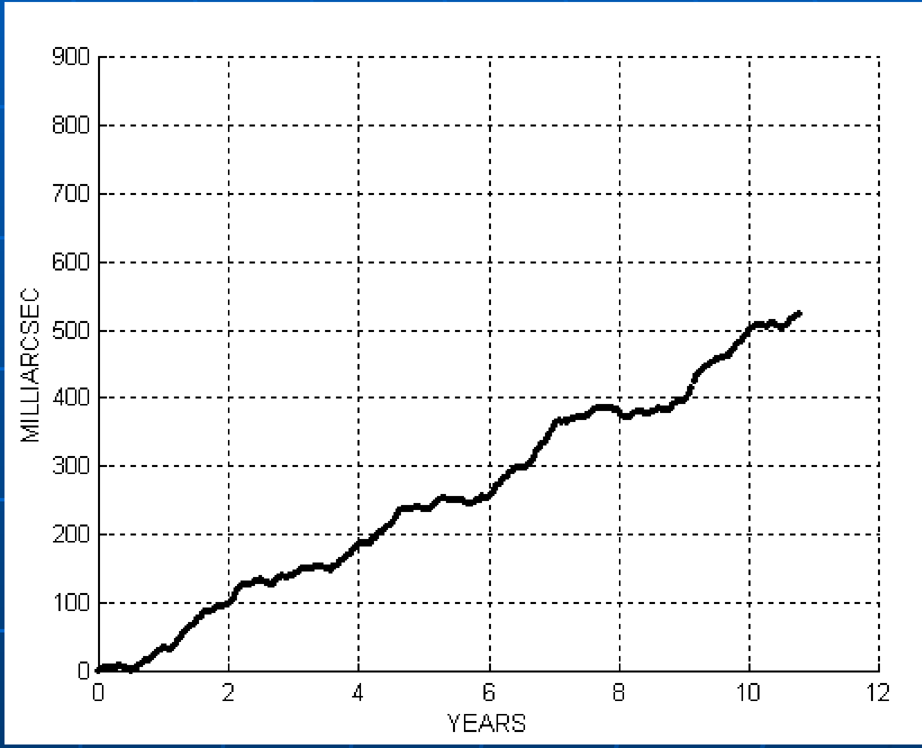
- Let us use the GFZ German orbital estimator EPOS (independent of GEODYN)

IC (Univ. Lecce), E. Pavlis (Univ Maryland Baltimore County),
R. Koenig and Neumayer (GFZ Munich/Potsdam),
G. Sindoni and A. Paolozzi (Univ. Roma I),
R. Matzner (Univ. Texas, Austin)

Using GEODYN (NASA) and EPOS (GFZ)

NEW 2006-2007 ANALYSIS OF THE LAGEOS ORBITS USING THE GFZ ORBITAL ESTIMATOR **EPOS**

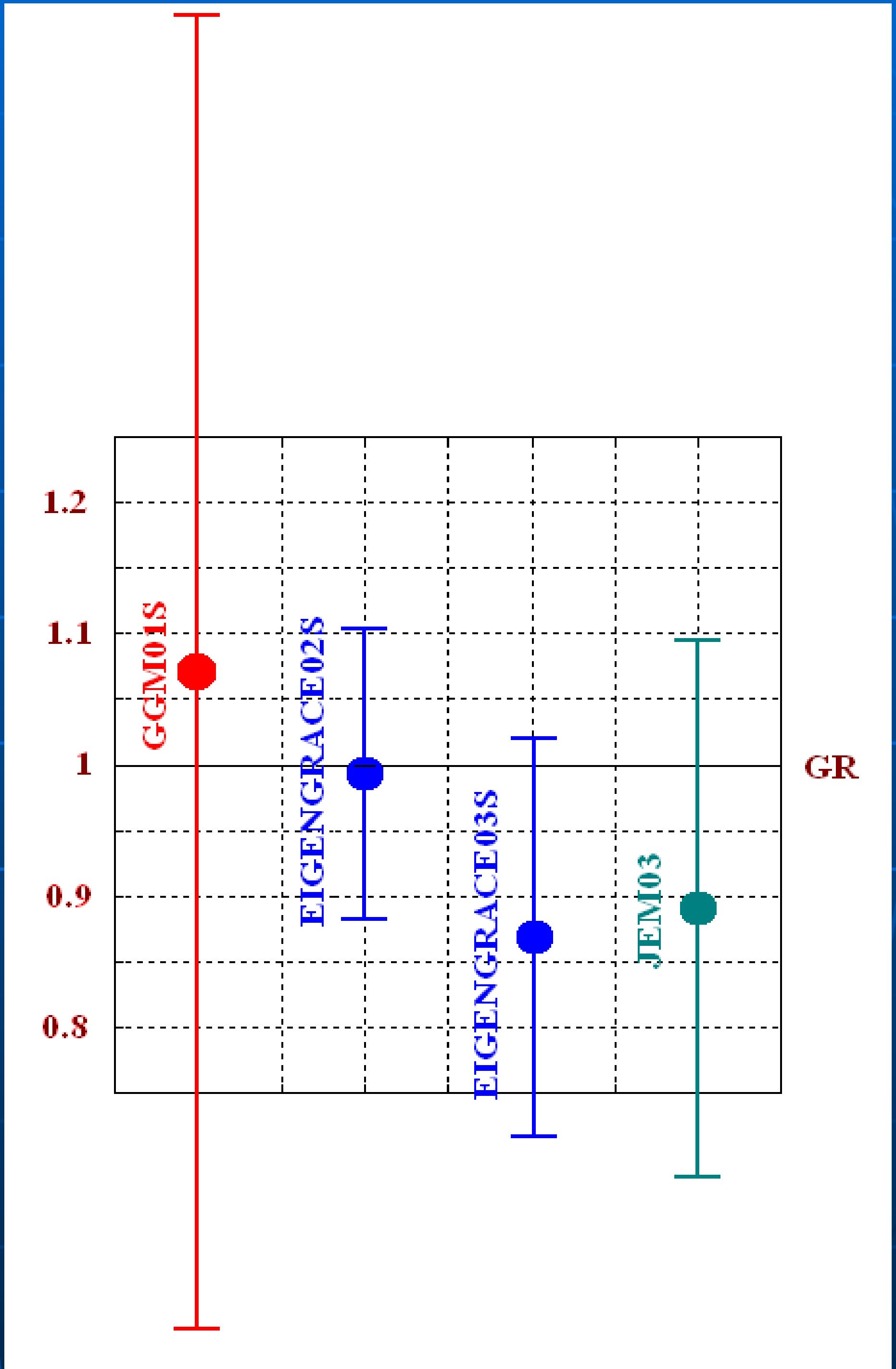


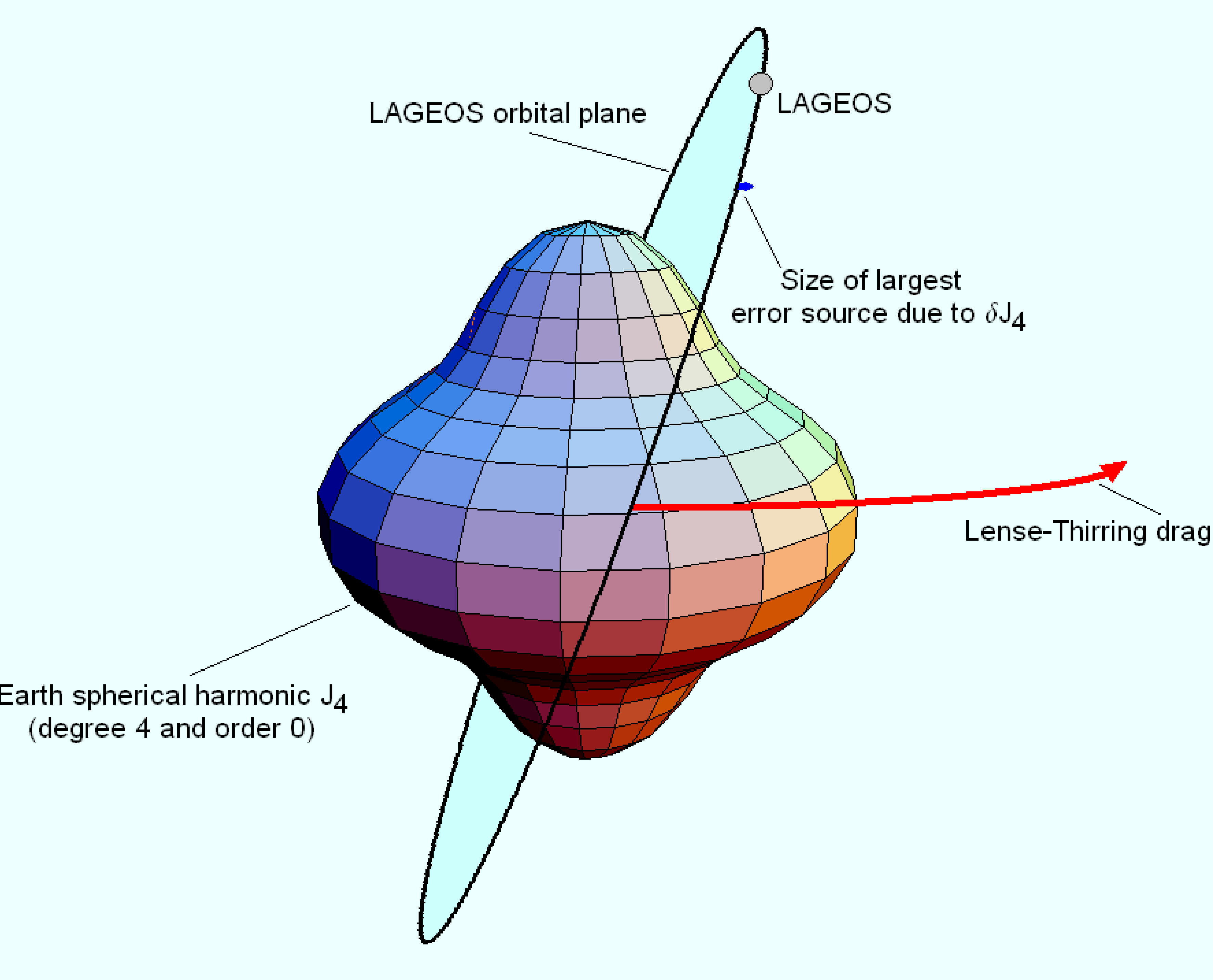


*by adding the geodetic precession of the orbital plane of an Earth satellite in the EPOS orbital estimator.

OLD 2004 ANALYSIS OF THE LAGEOS ORBITS USING THE NASA ORBITAL ESTIMATOR **GEODYN**

Comparison of Lense-Thirring effect measured using different Earth gravity field models

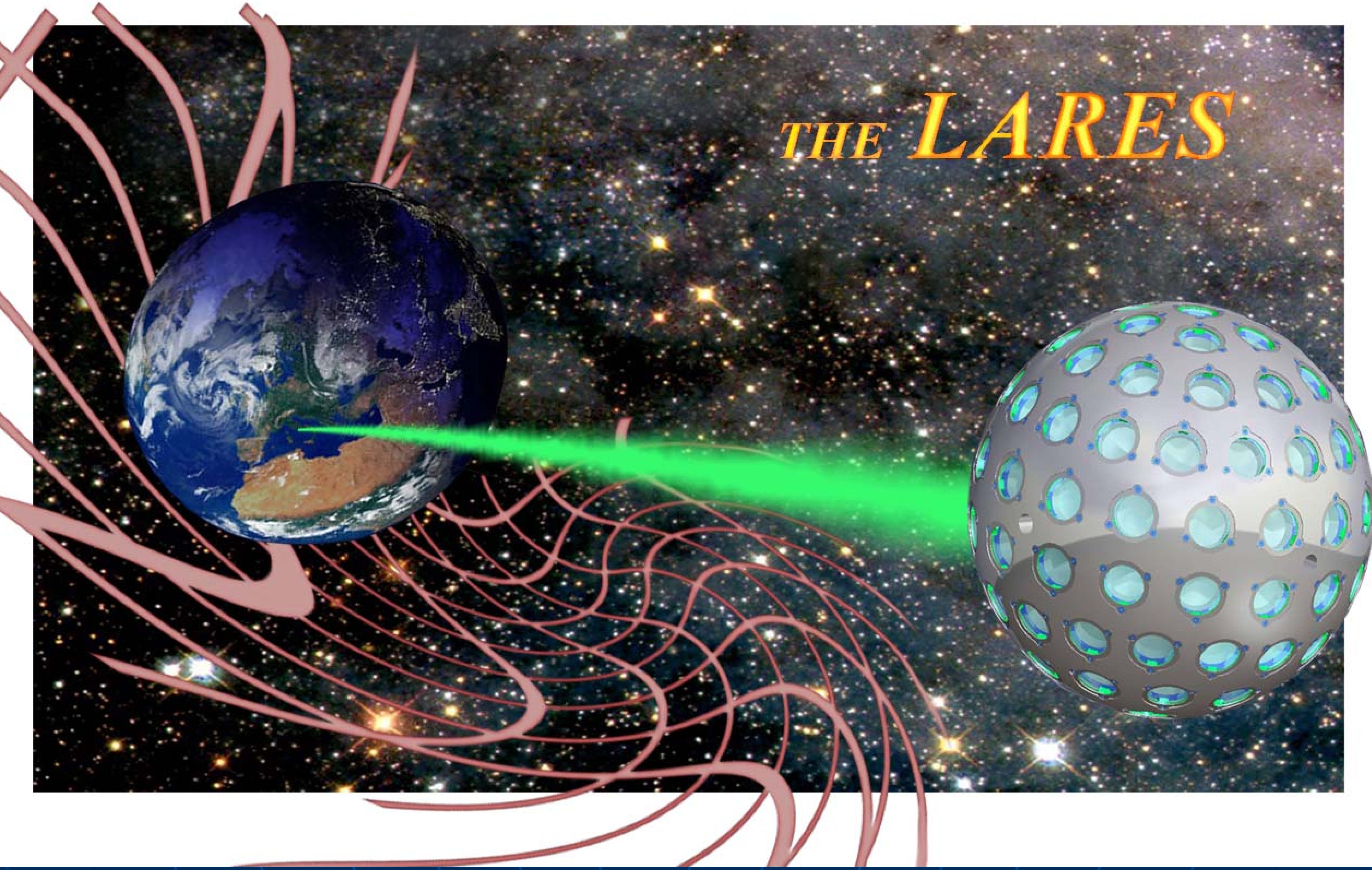


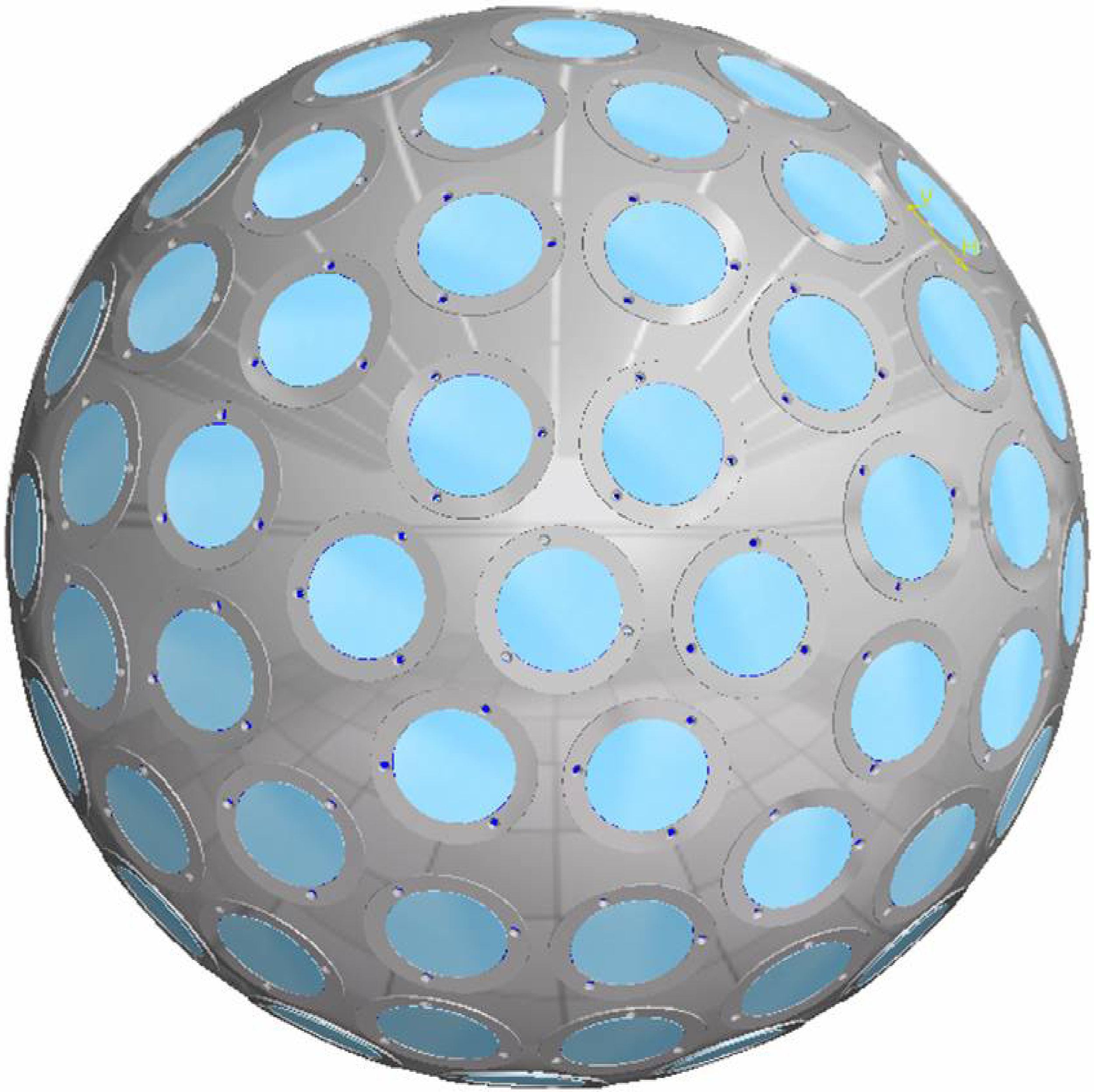


LAGEOS orbital plane
LAGEOS
Size of largest
error source due to δJ_4
Lense-Thirring drag
Earth spherical harmonic J_4
(degree 4 and order 0)

LARES

- Weight about 400 kg
- Radius about 18 cm
- Material Solid sphere of Tungsten alloy
- Semimajor Axis about 7900 km
- Eccentricity nearly zero
- Inclination about 71.5 degrees
- Combined with LAGEOS and LAGEOS 2 data it would provide a measurement of frame-dragging with accuracy of the order of 1 %


THE LARES




esa
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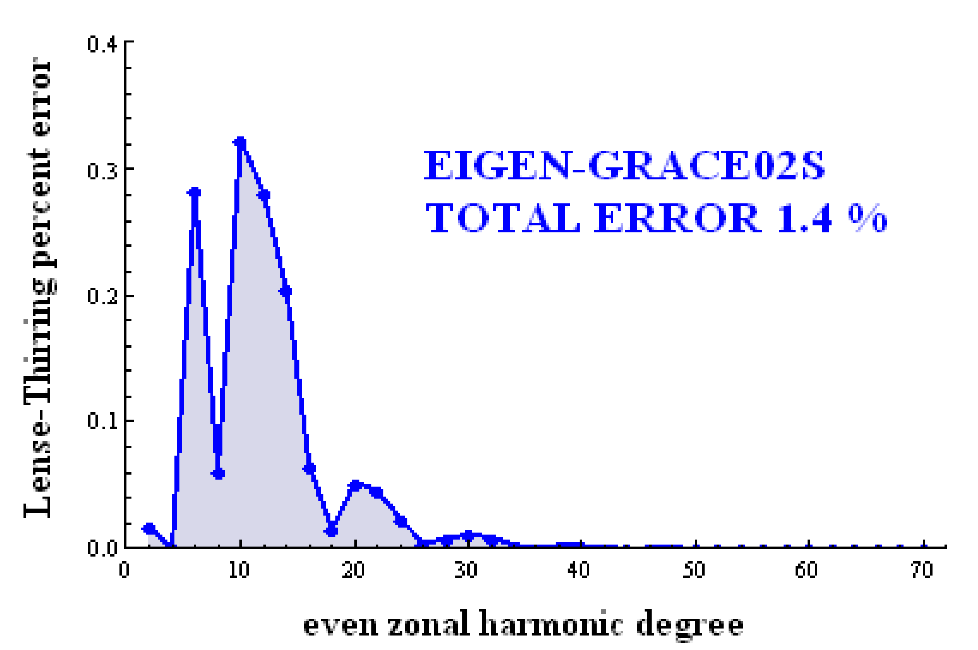
VEGA

GRAVITATIONAL ERRORS

Using the Earth gravitational model EIGEN-GRACE02S (February 2004), based on 111 days of GRACE observations, i.e., propagating the uncertainties of EIGEN-GRACE02S published by GFZ Potsdam on the nodes of LAGEOS, LAGEOS 2 and LARES and their combination, we find a total error of 1.4 %.

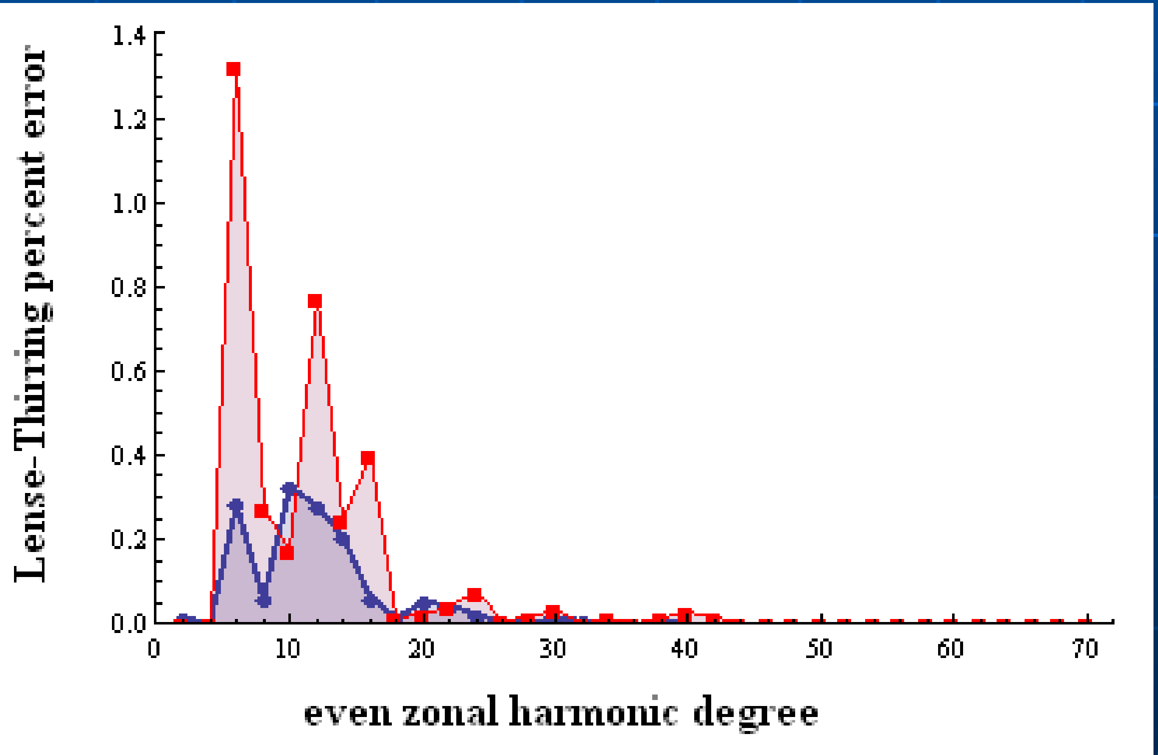
In particular we have calculated the error induced by the uncertainty of each even zonal harmonic up to degree 70: after degree 26 the error is negligible.



By the time of the LARES data analysis (2012-2015) we can assume an improvement in the GRACE Earth gravity field models of about one order of magnitude, thanks to much longer GRACE observations with respect to 111 days of EIGEN-GRACE02S and also to GOCE (2008).

GRAVITATIONAL ERRORS

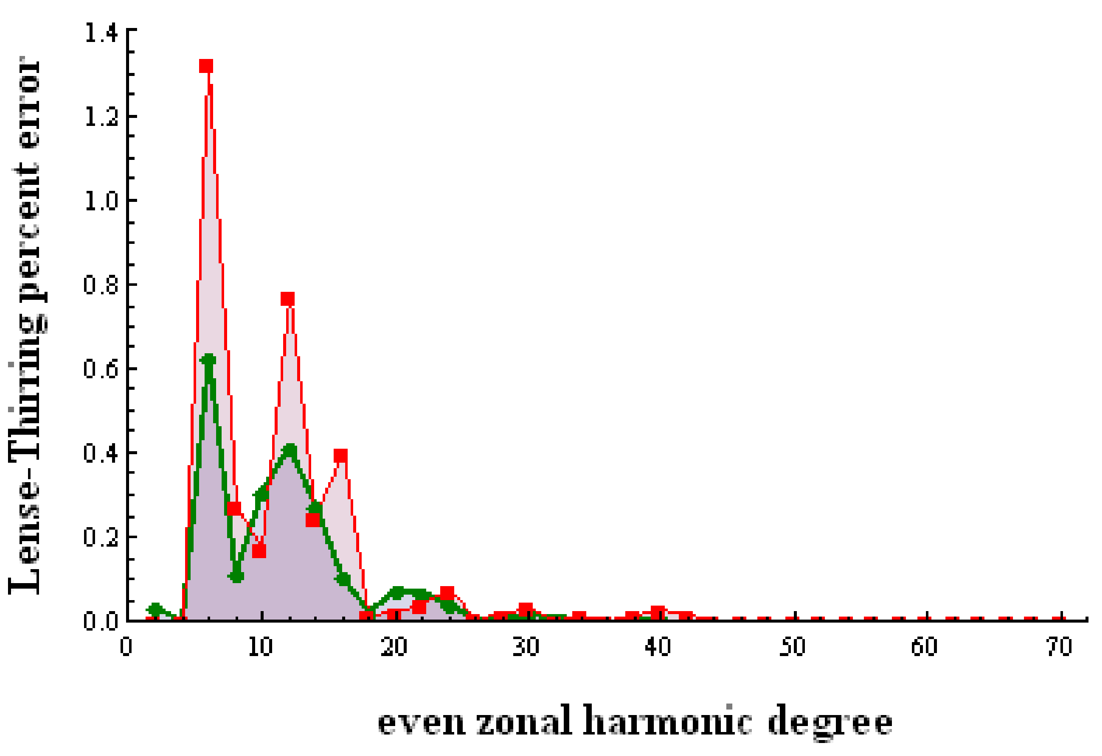
Standard technique in space geodesy to estimate the reliability of the published uncertainties of an Earth gravity model: take the difference between each harmonic coefficient of that model with the same harmonic coefficient of a diffeernt model and compare this difference with the published uncertainties. Let us take difference between each harmonic of the EIGEN-GRACE02S (GFZ Potsdam) model minus the same harmonic in the GGM02S (CSR Austin) model. CAVEAT: in order to use this technique, one must difference models of comparable accuracy, i.e., models that are indeed comparable, or use this method to only evaluate the less accurate model!



In Blue: percent errors in the measurement of the Lense-Thirring effect for EIGEN-GRACE02S for each even zonal

In Red: percent errors in the measurement of the Lense-Thirring effect using the difference between EIGEN-GRACE02S and GGM02S for each even zonal

GRAVITATIONAL ERRORS



In Green: percent errors in the measurement of the Lense-Thirring effect for GGM02S for each even Zonal harmonic

In Red: percent errors in the measurement of the Lense-Thirring effect Using the difference between EIGEN-GRACE02S and GGM02S for each even zonal harmonic

By the time of the LARES data analysis (2012-2015) we can assume an improvement in the GRACE Earth gravity field models of about one order of magnitude, thanks to much longer GRACE observations with respect to 111 days of EIGEN-GRACE02S and also to GOCE (2008).

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