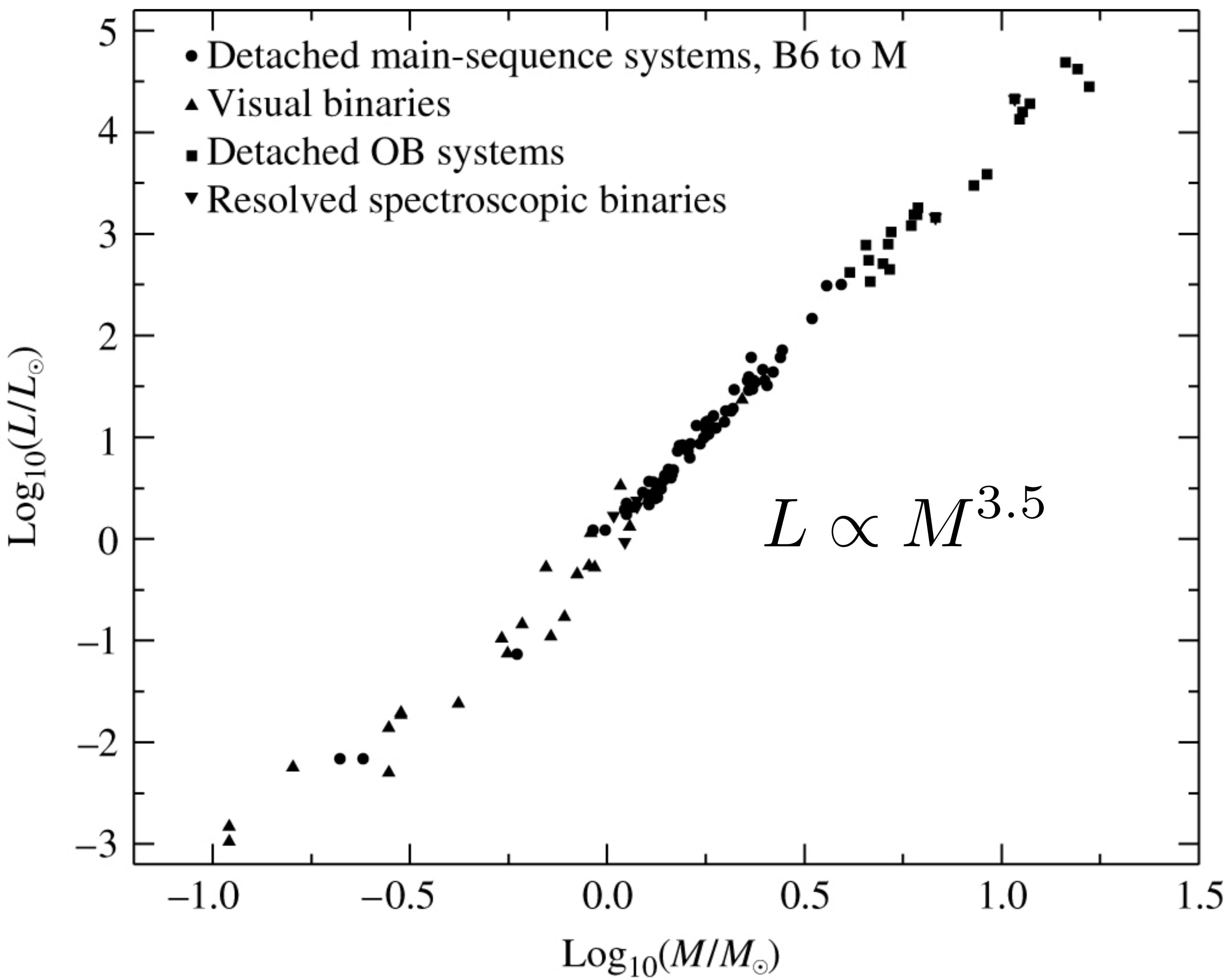
# Making sense of stars, part II

- Spectral classification

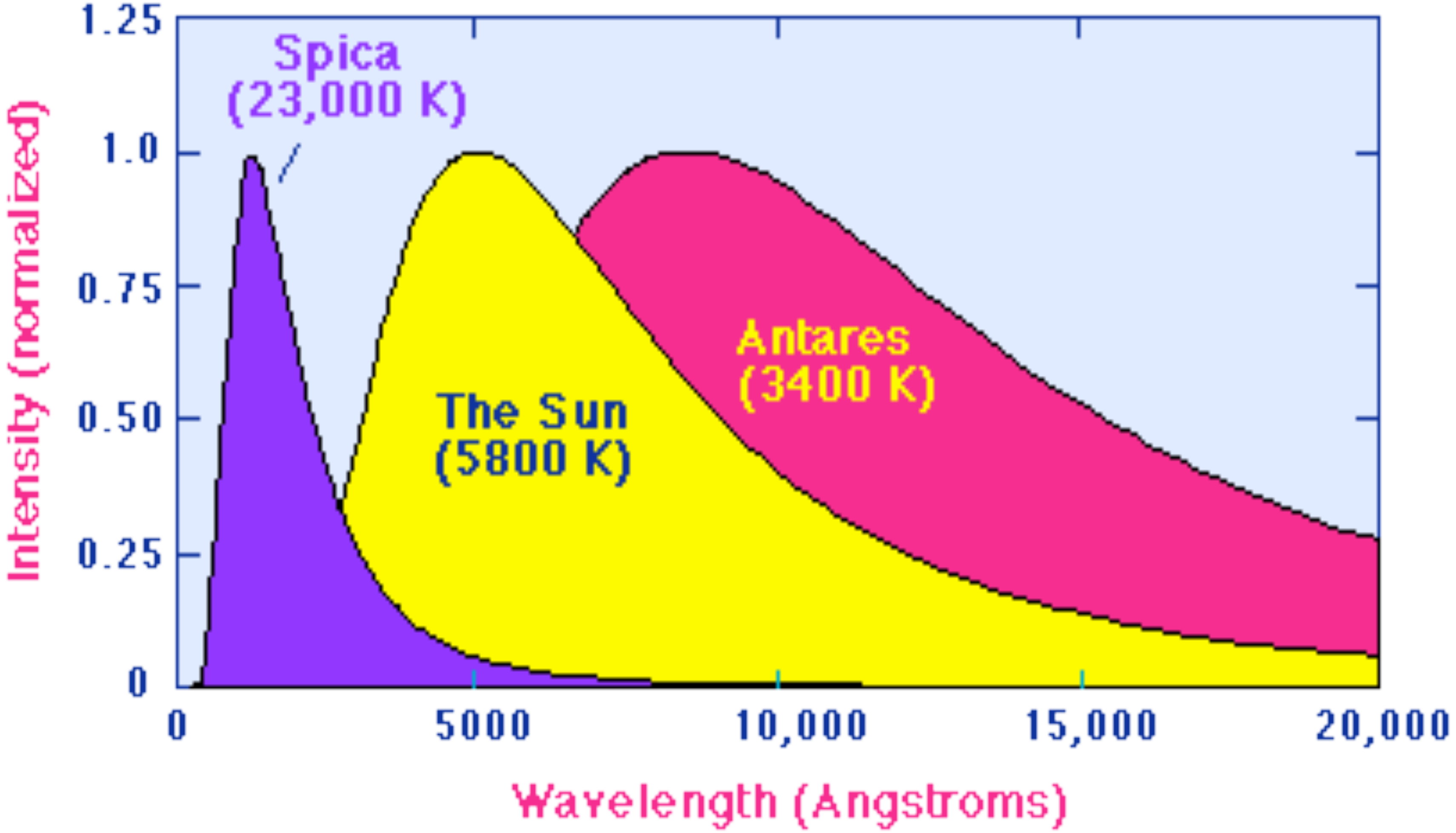
# We just found one piece of the puzzle: Mass luminosity relation




# Another piece is stellar spectra

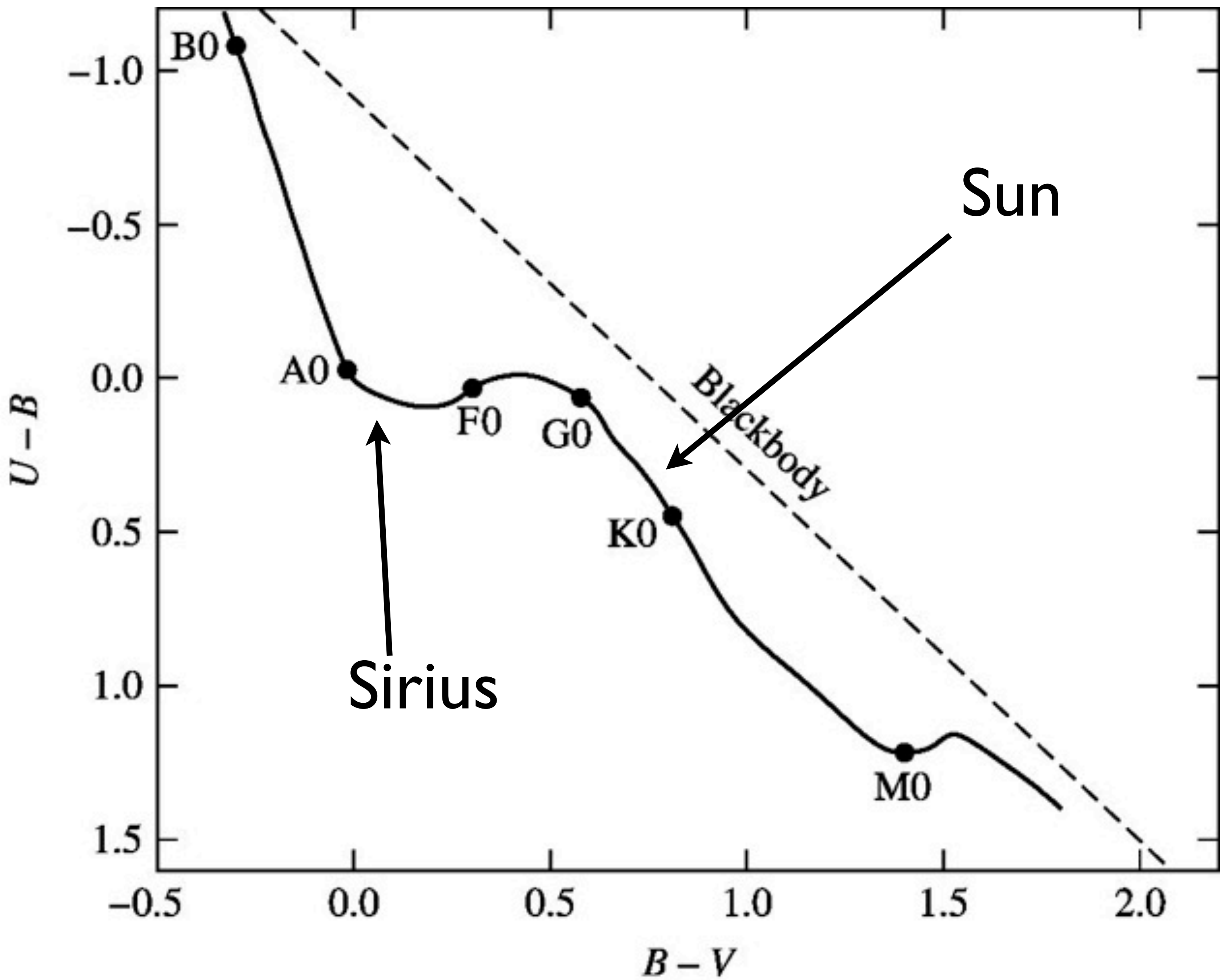
If stars were black bodies, we would easily measure their temperature.

$$\lambda(\mathbf{nm}) \approx \frac{\mathbf{3 \times 10^6}}{T(\mathbf{K})}$$

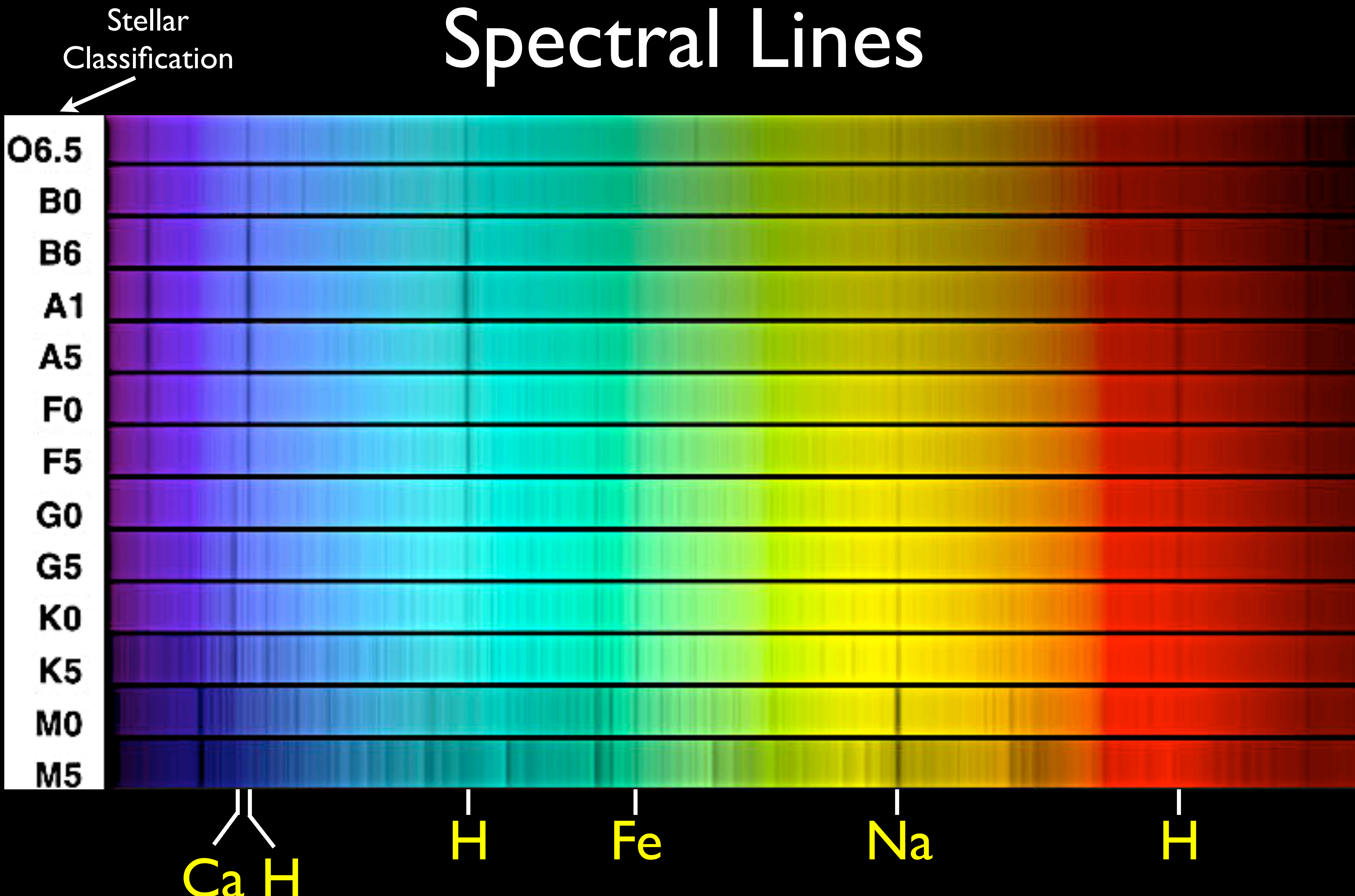



Slide

# But they are not quite



# Spectral Lines



# Spectral Classification of Stars

## Timeline:

1890s Edward C. Pickering (1846-1919) and Williamina P. Fleming (1857-1911) label spectra alphabetically according to strength of Hydrogen (Balmer) lines, beginning with “A” (strongest).

1890s Antonia Maury (1866-1952) developed a classification scheme based on the “width” of spectral lines. Would place “B” stars before “A” stars.

1901 Annie Cannon (1863-1941), brilliantly combined the above. Rearranged sequence, O before B before A, added decimal divisions (A0...A9) and consolidated classes. Led to classification scheme *still used by astronomers today!*

OBAFGKM (*Oh Be A Fine Guy/Girl, Kiss Me)*

“Early Type” Stars

“Late Type” Stars

# Spectral Classification of Stars

## Timeline:

1901 Annie Cannon (1863-1941), brilliantly combined the above. Rearranged sequence, O before B before A, added decimal divisions (A0...A9) and consolidated classes. Led to classification scheme *still used by astronomers today!*

OBAFGKM (*Oh Be A Fine Guy/Girl, Kiss Me)*

"Early Type" Stars : Stars near the beginning of Sequence

"Late Type" Stars : Stars near the end of the Sequence.

One can mix the definitions: K0 star is an "early-type" K star. B9 is a "late-type" B star.

1911-1914 Annie Cannon classified 200,000 spectra, listed in the Henry Draper Catalog. Catalog ID's are "HD 39801" (ID for Betelgeuse in the constellation Orion).

During 1990s Two new letters added to Sequence for very cool, Brown-Dwarf stars. "L" spectral types (T=1300-2500 K) and "T" types (T < 1300 K).

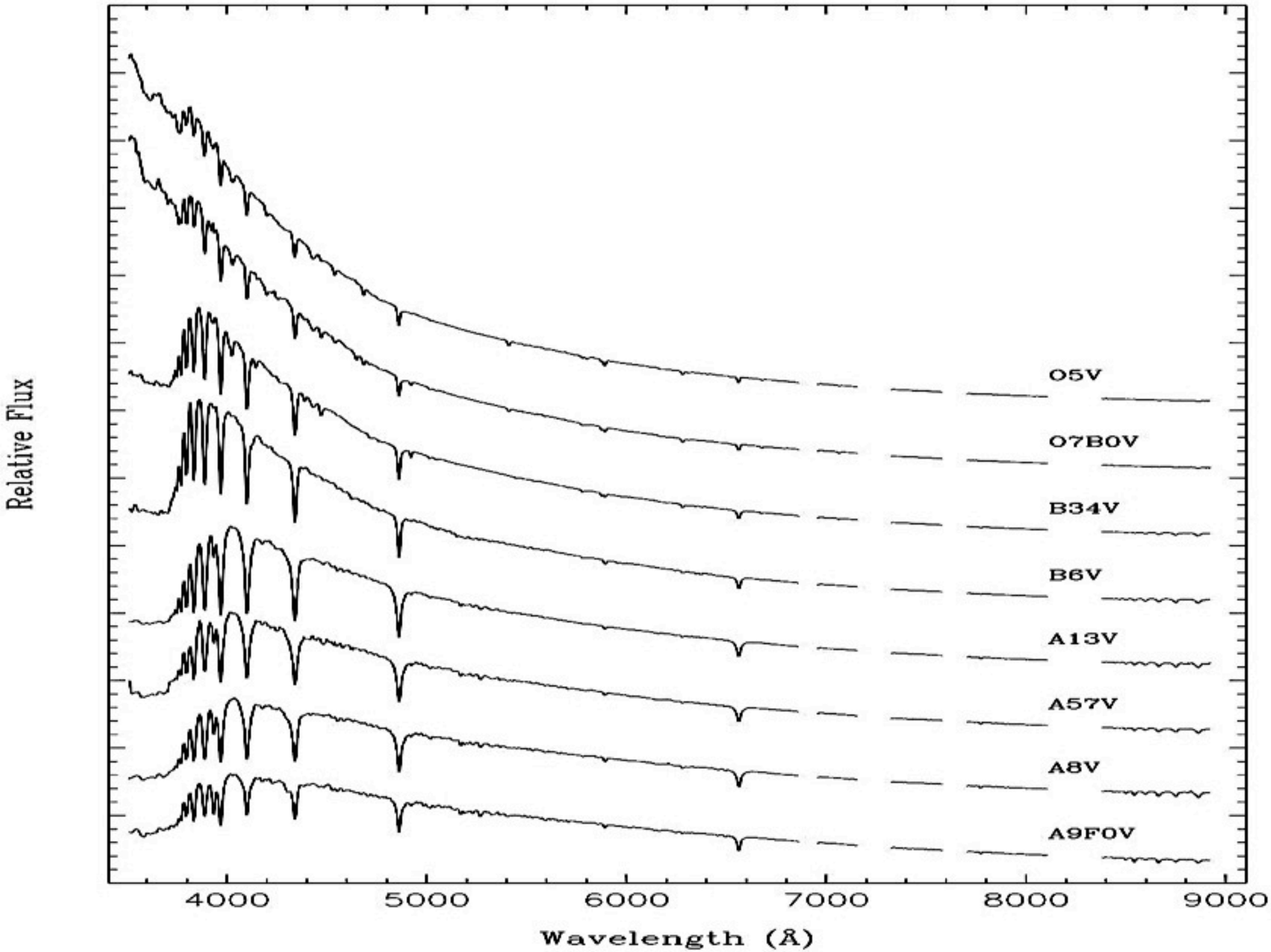
OBAFGKMLT (*Oh Be A Fine Guy/Girl, Kiss Me - Less Talk !)*

# Spectral Classification of Stars

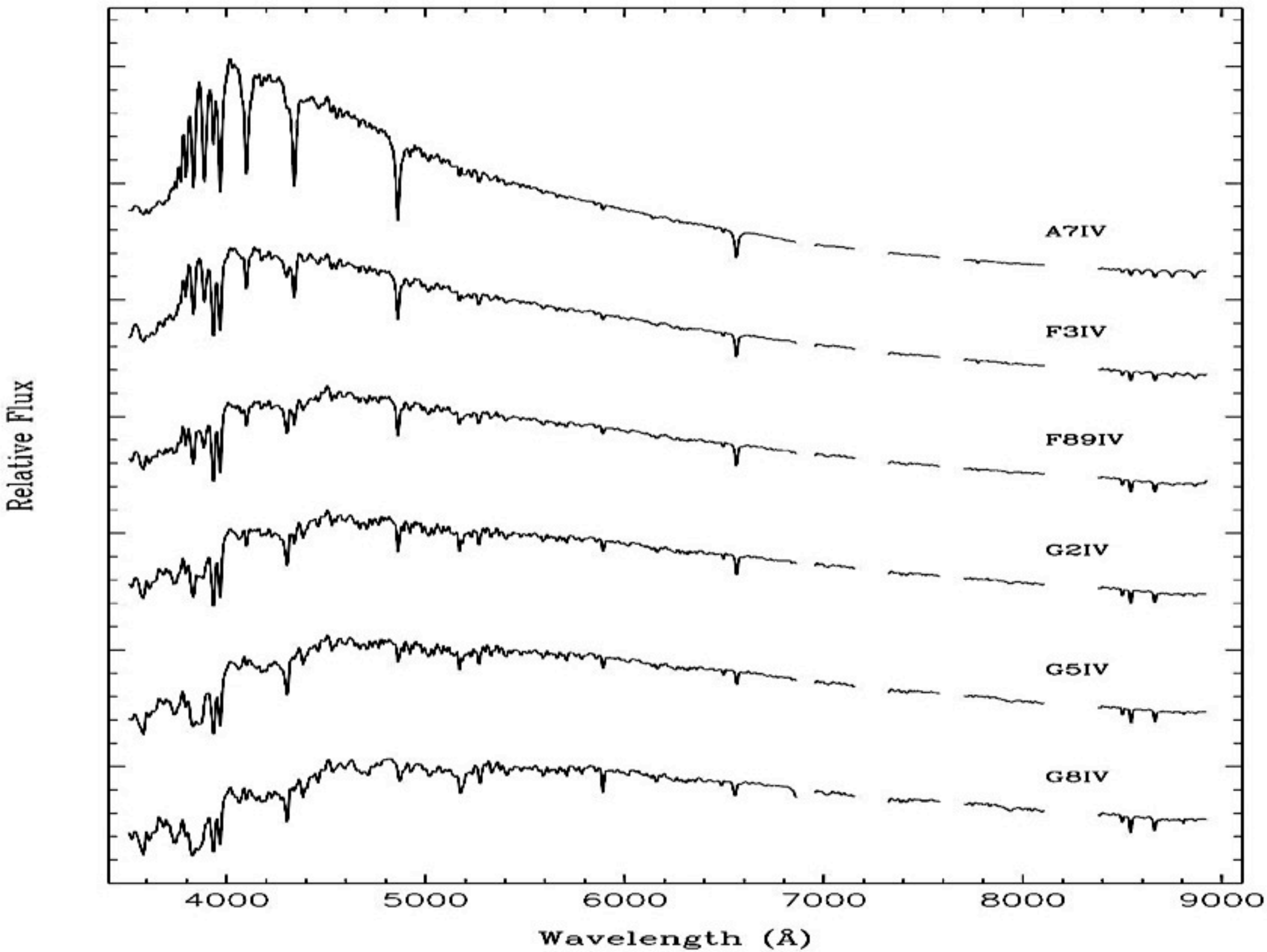
Hotter

Cooler

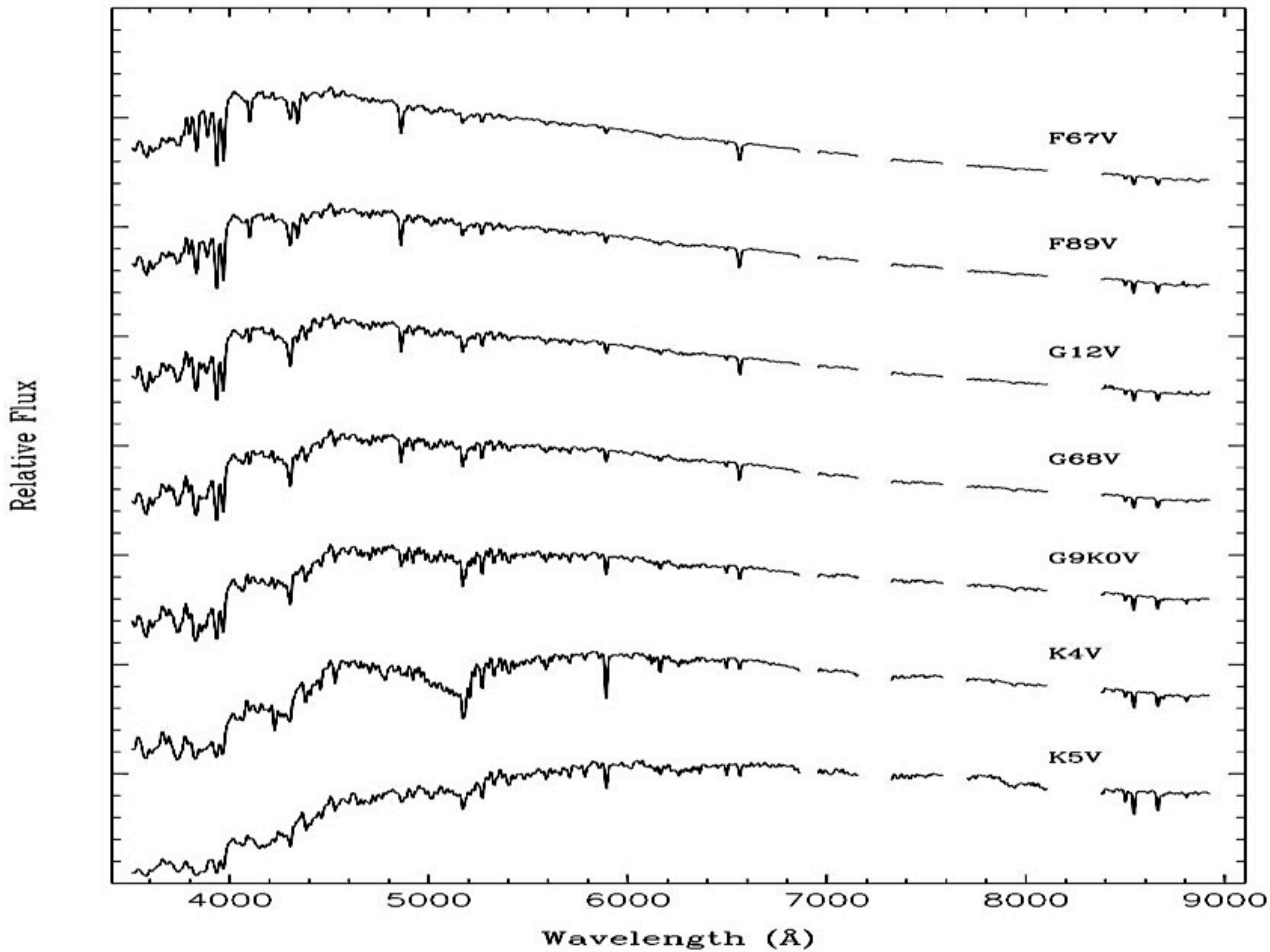
| Spectral Type | Characteristics |
|---|---|
| O | Hottest blue-white stars, few lines. Strong He II ($He^+$) absorption lines. He I (neutral helium) stronger). |
| B | Hot blue-white. He I (neutral Helium), strongest at B2. H I (neutral Hydrogen) stronger. |
| A | White stars. Balmer absorption lines strongest at A0 (Vega), weaker in later-type A stars. Strong Ca II ($Ca^+$) lines. |
| F | Yellow-white stars. Ca II lines strengthen to later types. F-stars. Balmer lines strengthen to earlier type F-stars. |
| G | Yellow stars (Sun is a G5 star). Ca II lines become stronger. Fe I (neutral iron) lines become strong. |
| K | Cool orange stars. Ca II (H and K) lines strongest at K0, becoming weaker in later stars. Spectra dominated by metal absorption lines. |
| M | Cool red stars. Spectra dominated by *molecular* absorption bands, e.g., TiO (titanium oxide). Neutral metal lines strong. |
| L | Very cool, dark red (brown dwarfs). Brighter in Infrared than visible. Strong molecular absorption bands, e.g., CrH, FeH, water, CO. TiO weakening. |
| T | Coolest stars. Strong methane ($CH_4$), weakening CO bands. |



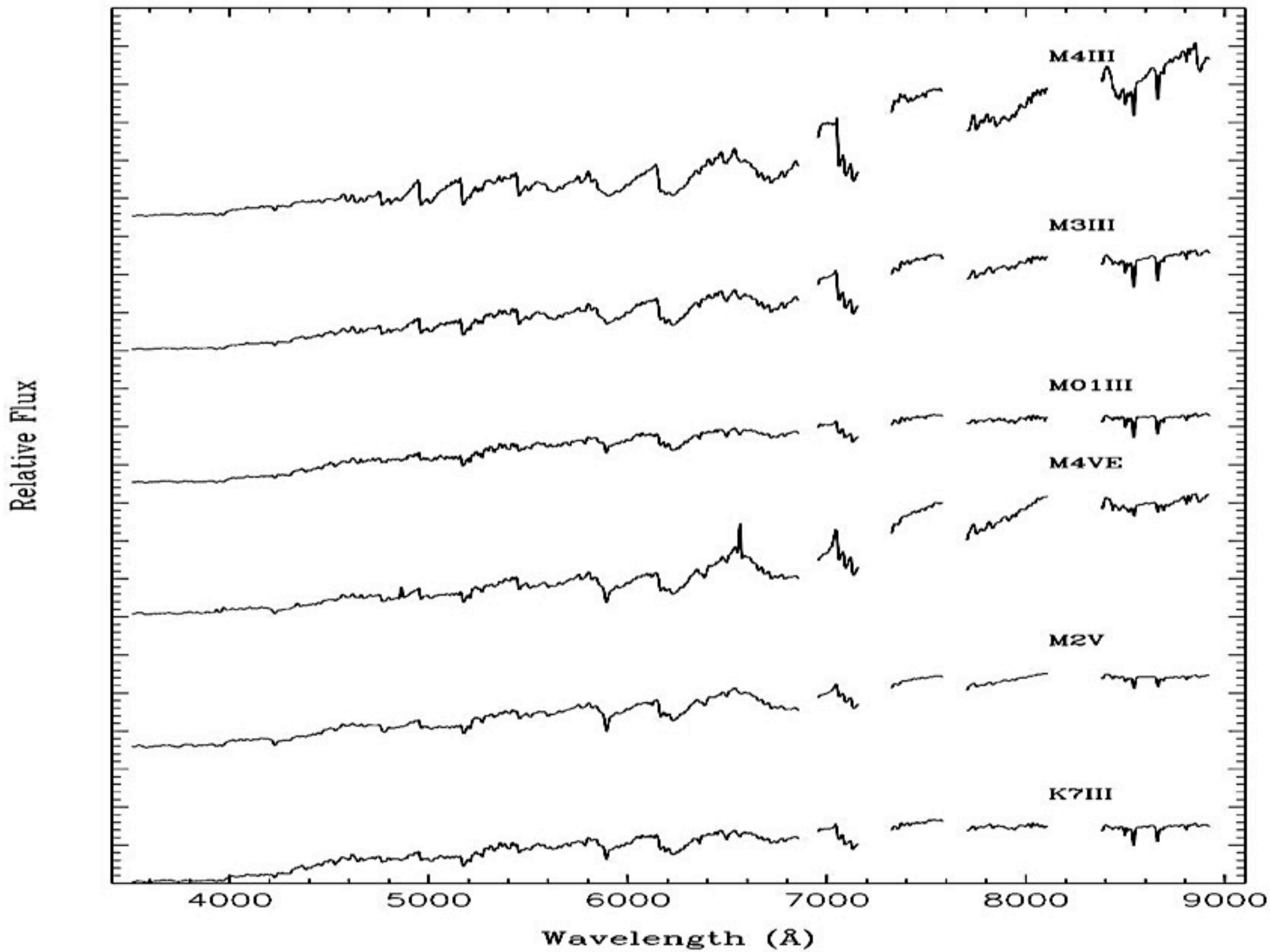
Relative Flux
Wavelength (Å)
4000
5000
6000
7000
8000
9000
O5V
O7B0V
B34V
B6V
A13V
A57V
A8V
A9F0V

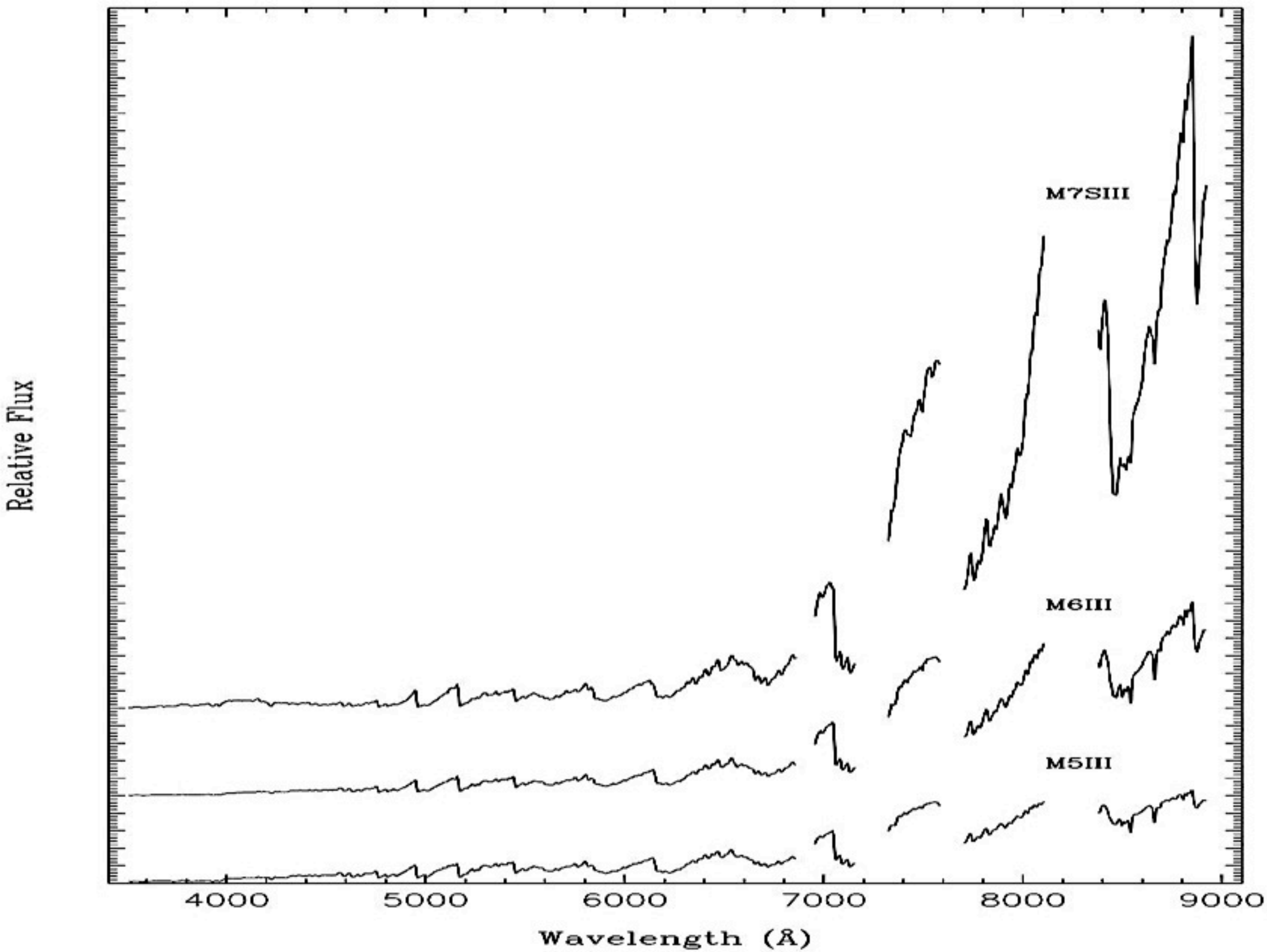


Relative Flux
A7IV
F3IV
F89IV
G2IV
G5IV
G8IV
4000
5000
6000
7000
8000
9000
Wavelength (Å)



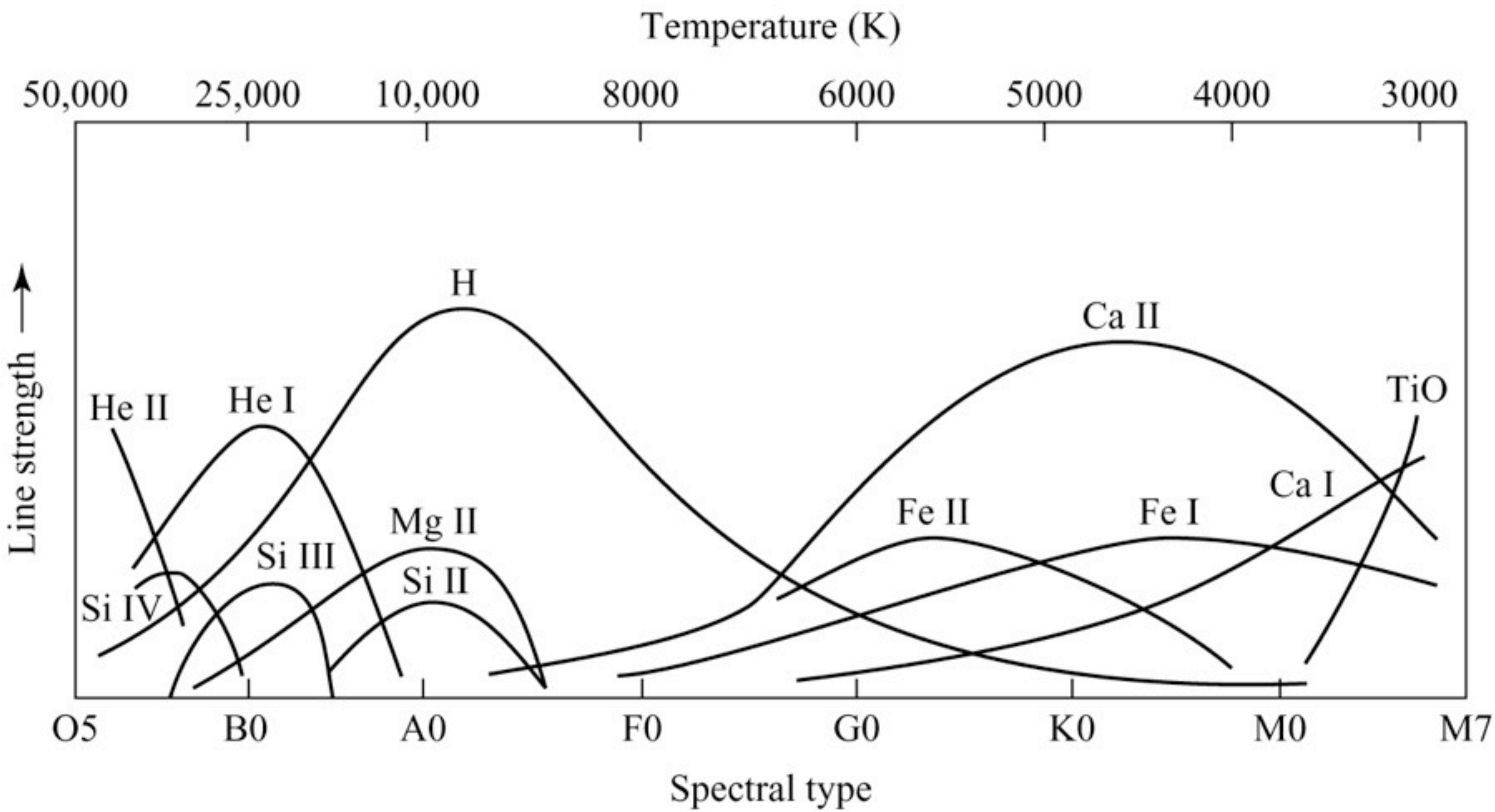
F67V
F89V
G12V
G68V
G9K0V
K4V
K5V
4000
5000
6000
7000
8000
9000
Wavelength (Å)
Relative Flux


M4III
M3III
M01III
M4VE
M2V
K7III
Relative Flux
4000
5000
6000
7000
8000
9000
Wavelength (Å)


M7SIII
M6III
M5III
4000
5000
6000
7000
8000
9000
Wavelength (Å)
Relative Flux

# Spectral Classification of Stars

Physical explanation? Maybe different chemical composition?



# Similar composition of atmospheres for 90% of the stars

**TABLE 6-2**
The Most Abundant Elements in the Sun

| Element | Percentage by Number of Atoms | Percentage by Mass |
|---|---|---|
| Hydrogen | 91.0 | 70.9 |
| Helium | 8.9 | 27.4 |
| Carbon | 0.03 | 0.3 |
| Nitrogen | 0.008 | 0.1 |
| Oxygen | 0.07 | 0.8 |
| Neon | 0.01 | 0.2 |
| Magnesium | 0.003 | 0.06 |
| Silicon | 0.003 | 0.07 |
| Sulfur | 0.002 | 0.04 |
| Iron | 0.003 | 0.1 |

# Spectral Classification of Stars

## Physical Description

Stars are not composed of pure hydrogen, but nearly all atoms (mostly H, He, and *metals* = anything *not* H or He).

Typically 1 He atom for every 10 H atoms (and even fewer *metals*).

Helium (and metals) provide more electrons, which can recombine with ionized H.

So, it takes *higher* temperatures to achieve same degree of H ionization when He and metals are present.

Abundance is = $\log_{10}(N_{element}/N_H) + 12$.

I.e., Abudance of Oxygen = 8.83, which means:

$8.83 = \log10(N_O/N_H) + 12$

$N_O/N_H = 10^{8.83 - 12} = 0.000676 \approx 1/1480$

There is one Oxygen atom
for every 1480 H atoms !

Most Abundant Elements in the Solar Photosphere.

| Element | Atomic # | Log Relative Abundance |
|---|---|---|
| H | 1 | 12.00 |
| He | 2 | 10.93 ± 0.004 |
| O | 8 | 8.83 ± 0.06 |
| C | 6 | 8.52 ± 0.06 |
| Ne | 10 | 8.08 ± 0.06 |
| N | 7 | 7.92 ± 0.06 |
| Mg | 12 | 7.58 ± 0.05 |
| Si | 14 | 7.55 ± 0.05 |
| Fe | 26 | 7.50 ± 0.05 |
| S | 16 | 7.33 ± 0.11 |
| Al | 13 | 6.47 ± 0.07 |
| Ar | 18 | 6.40 ± 0.06 |
| Ca | 20 | 6.36 ± 0.02 |
| Ng | 11 | 6.33 ± 0.03 |
| Ni | 28 | 6.25 ± 0.04 |

# Spectral Classification of Stars
## Physical Description

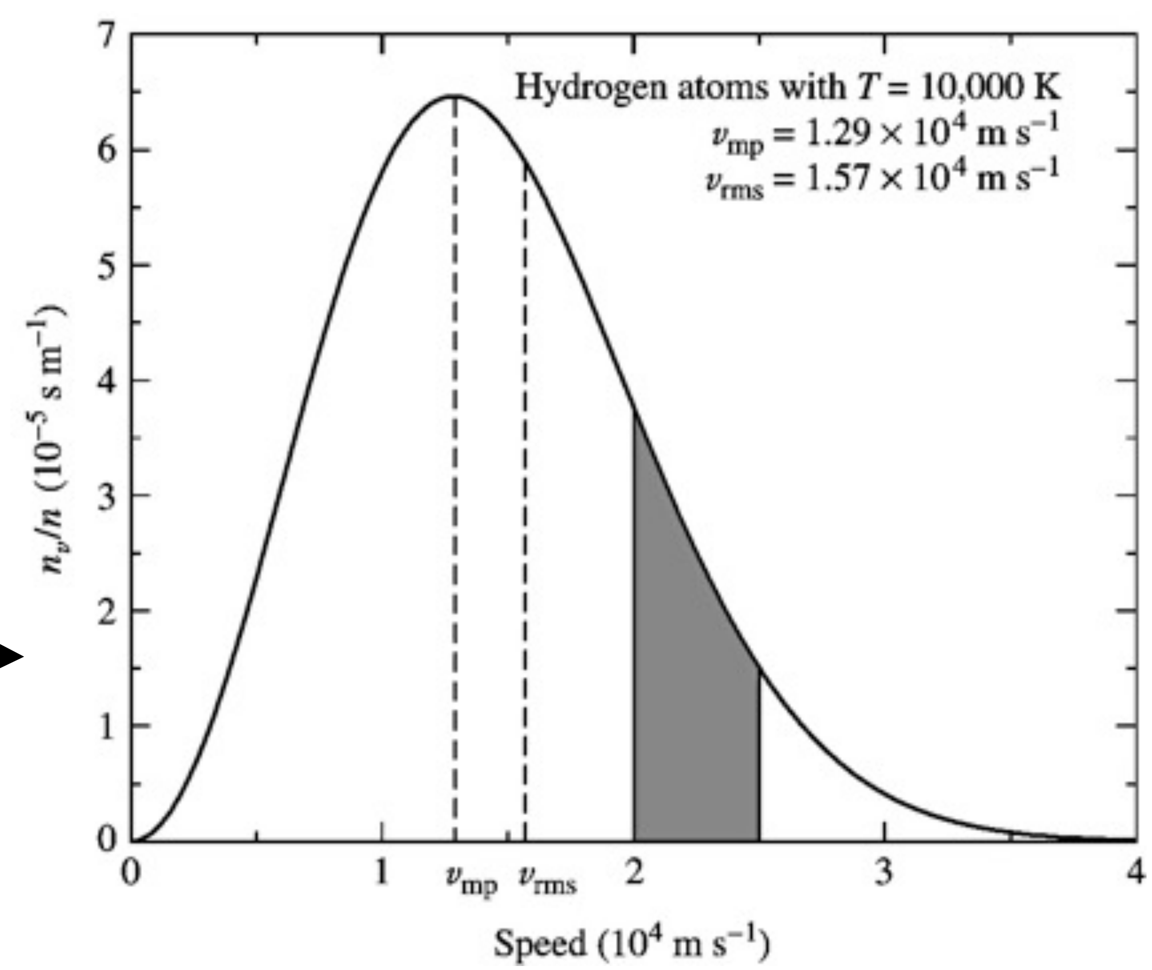
Need to use *Statistical Mechanics* to understand the physical state and conditions of the huge number of atoms (and molecules) in the atmospheres of stars.

**Maxwell-Boltzmann** velocity distribution function, number of particles with temperature T per unit volume having speeds between v and v+dv:

$$n_v\, dv = n \left[ m / (2\pi kT) \right]^{3/2} \exp( -mv^2/2kT ) \times 4\pi v^2\, dv$$

n is the total number density (particles per unit volume),
$n_v = \partial n/\partial v$,
m is the particle mass.

Maxwell-Boltzmann distribution for hydrogren at T=10,000 K



# Spectral Classification of Stars
## Physical Description

**Maxwell-Boltzmann** velocity distribution function, number of particles with temperature T per unit volume having speeds between v and v+dv:

$$n_v\, dv = n \left[ m / (2\pi kT) \right]^{3/2} \exp( -mv^2/2kT ) \times 4\pi v^2\, dv$$

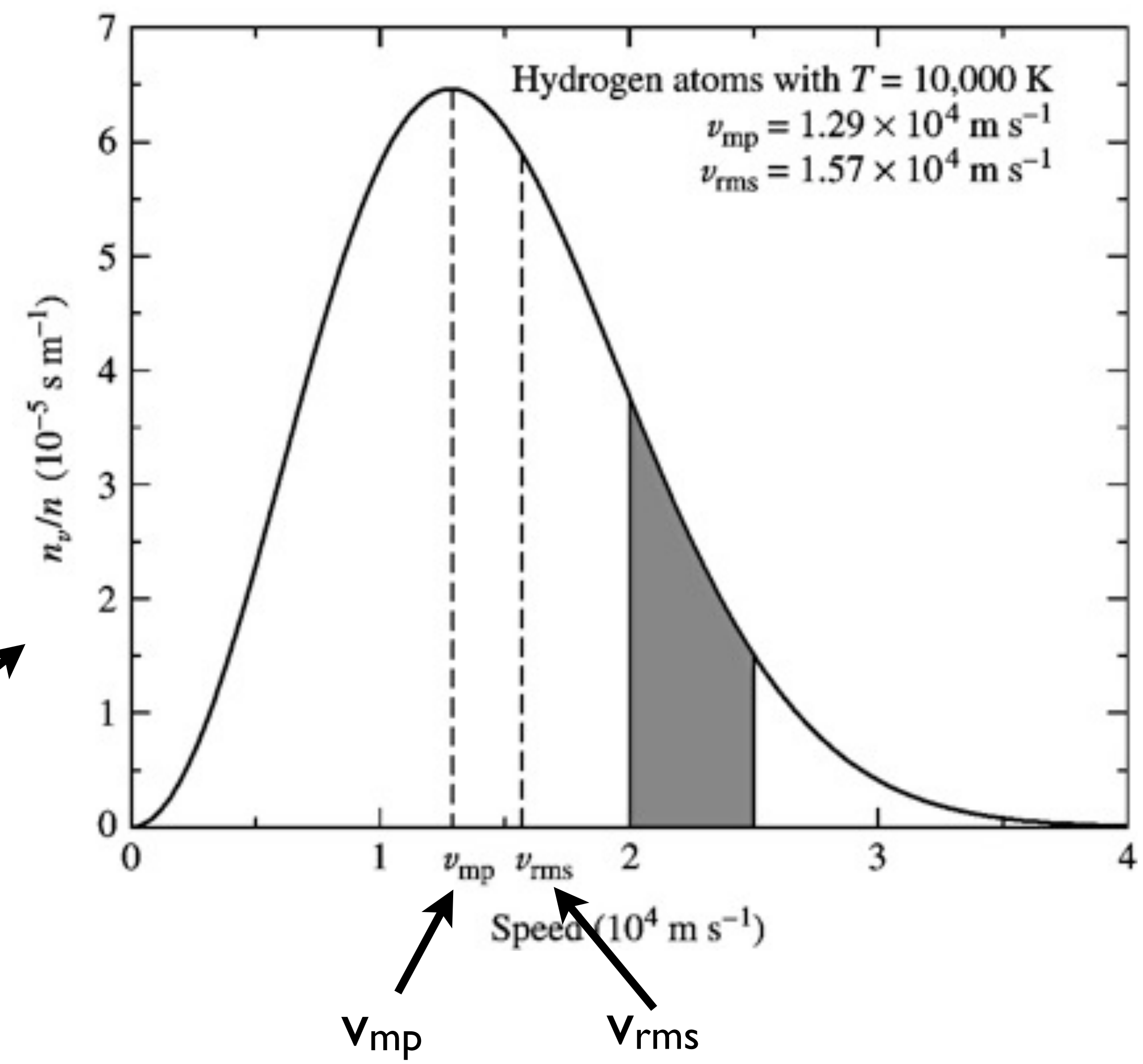
**Most probable speed (mode)** comes for $d(n_v)/dv = 0$, which gives:

$$v_{mp} = \sqrt{2kT / m}$$

Average, **Root-mean-squared (rms) speed** is:

$$v_{rms} = \sqrt{3kT / m}$$

Maxwell-Boltzmann distribution for hydrogen at T=10,000 K



# Spectral Classification of Stars
## Physical Description

**Boltzmann Equation**: Distribution of electrons in atomic orbital levels. General result: orbitals of higher energy are less likely to be occupied by electrons.

$s_a$ = set of quantum numbers of state with energy $E_a$.
$s_b$ = set of quantum numbers of state with energy $E_b$.

$$\frac{P(s_b)}{P(s_a)} = \frac{\exp(-E_b/2kT)}{\exp(-E_a/2kT)} = \exp(-[E_b-Ea]/2kT)$$

The above is the ratio of the probability that the system is in state $s_b$ to the probability that it is in state $s_a$. Term exp(-E/kT) is the *Boltzmann factor*.

Example: Hydrogen in ground state, $E_a$ = -13.6 eV corresponds to $s_a = \{n=1, \ell=0, m_\ell=0, m_s = +1/2\}$.

Limits: Consider $E_b > E_a$, energy of state $s_b$ is greater than state $s_a$.

As T goes to 0, $-[E_b-Ea]/2kT$ goes to minus infinity, and $P(s_b)/P(s_a)$ goes to zero.

As T goes to infinity, $-[E_b-Ea]/2kT$ goes to zero, and $P(s_b)/P(s_a)$ goes to one (all atomic energy levels available with equal probability).

# Spectral Classification of Stars
## Physical Description

In most cases, energy levels may be **degenerate**, where more than one quantum state has same energy. If $s_a$ and $s_b$ are degenerate then $E_a=E_b$, but $s_a \neq s_b$.

Define $g_b$ to be the number of states with energy $E_b$. $g_b$ is the **statistical weight** of the energy level.

**Example: Hydrogen**

Ground state has a twofold degeneracy. That is, there are **two** states with different quantum numbers that have the same energy, $E_1$=-13.6 eV. $g_1$=2.

The first excited state has $E_2$=-3.40 eV. There are **eight** quantum states with this energy. Therefore, $g_2$ = 8.

| **Ground states $s_1$** | | | | **Energy $E_1$** |
|---|---|---|---|---|
| n | $\ell$ | $m_\ell$ | $m_s$ | (eV) |
| 1 | 0 | 0 | +1/2 | -13.6 |
| 1 | 0 | 0 | -1/2 | -13.6 |
| | | | | |
| **First Excited States $s_2$** | | | | **Energy $E_2$** |
| n | $\ell$ | $m_\ell$ | $m_s$ | (eV) |
| 2 | 0 | 0 | +1/2 | -3.40 |
| 2 | 0 | 0 | -1/2 | -3.40 |
| 2 | 1 | 1 | +1/2 | -3.40 |
| 2 | 1 | 1 | -1/2 | -3.40 |
| 2 | 1 | 0 | +1/2 | -3.40 |
| 2 | 1 | 0 | -1/2 | -3.40 |
| 2 | 1 | -1 | +1/2 | -3.40 |
| 2 | 1 | -1 | -1/2 | -3.40 |

# Spectral Classification of Stars
## Physical Description

$$\frac{P(E_b)}{P(E_a)} = \frac{g_b \exp( -E_b/kT )}{g_a \exp( -E_a/kT )} = (g_b / g_a) \exp( -[E_b-E_a]/kT )$$

Thus, for the atoms of a given element in a specified state of ionization, the ratio of the # of atoms $N_b$ with energy $E_b$ to the number of atoms $N_a$ with energy $E_a$ *in different states of excitation* is given by the **Boltzmann Equation.**

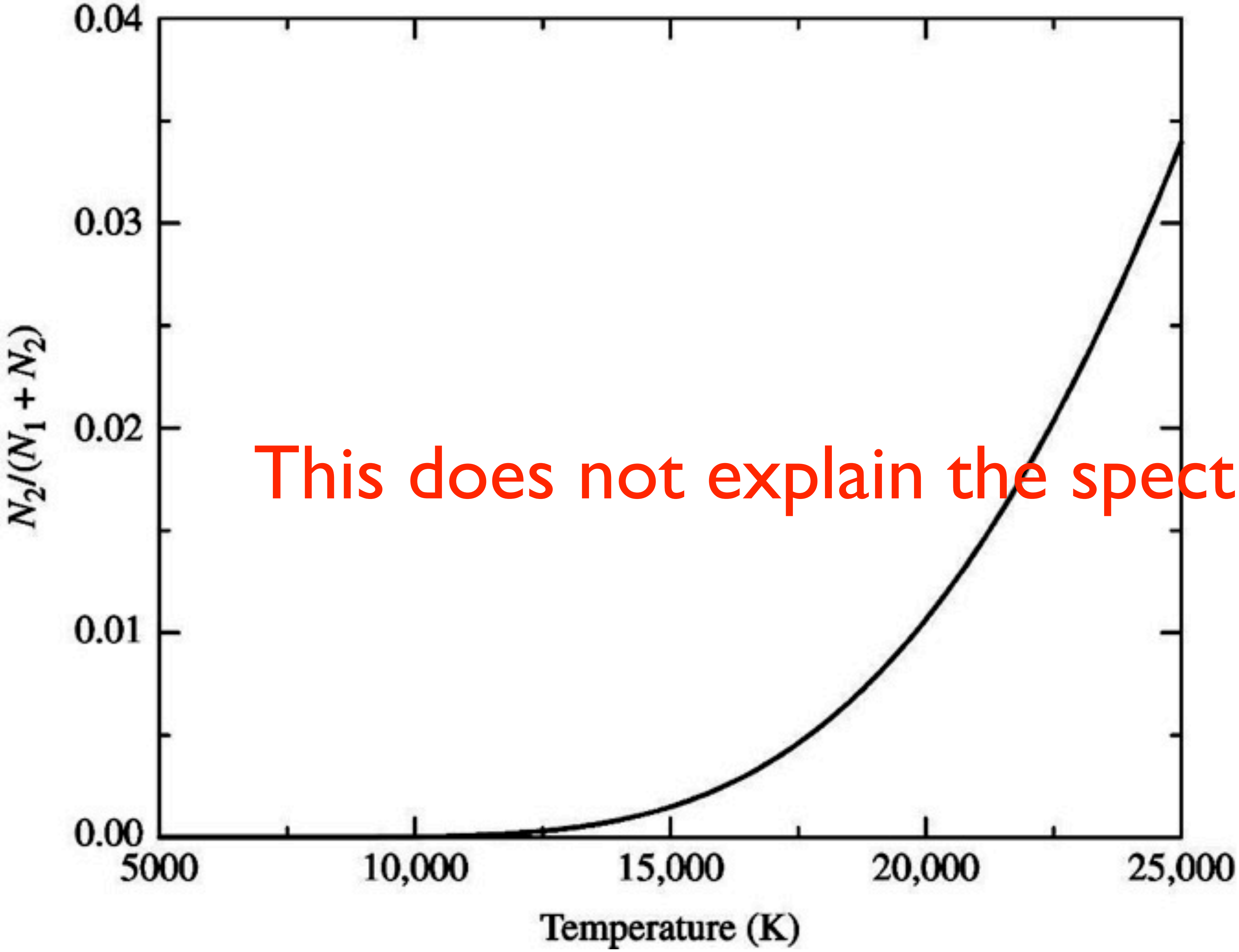
$$\frac{N_b}{N_a} = \frac{g_b \exp( -E_b/kT )}{g_a \exp( -E_a/kT )} = (g_b / g_a) \exp( -[E_b-E_a]/kT )$$

Example: For a gas of hydrogen atoms, at what temperature will equal numbers of atoms have electrons in the ground state (n=1) and in the first excited state (n=2). For hydrogen, the degeneracy is $g_n = 2n^2$ ($g_1=2$, $g_2=8$, $g_3=18$, ...). Setting $N_2=N_1$ in the Boltzmann equation gives,

$$1 = (g_b / g_a) \exp( -[E_b-E_a]/kT ) = (8/2) \exp( -[-3.40 \text{ eV} - (-13.6\text{eV})]/kT )$$

$$10.2 \text{ eV} / kT = \ln (4) \quad \text{or } T = 85{,}000 \text{ K !}$$

# Spectral Classification of Stars
## Physical Description




This does not explain the spectra!

Ratio of the number of hydrogen atoms in the first excited state ($N_2$) to the total number of hydrogen atoms ($N_1 + N_2$) using the Boltzmann equation.

# Spectral Classification of Stars
## Physical Description

**Saha Equation**: Need to incorporate the relative number of atoms in *different stages of ionization*.

Let $\chi_i$ be the ionization energy needed to remove an electron from an atom (or ion). For example, to convert neutral hydrogen (H I = $H^0$) to ionized hydrogen (H II =$H^+$) you can have $\chi_i$ = 13.6 eV for hydrogen in the ground state, or $\chi_i$=3.40 eV for hydrogen in the first excited state, etc.

Average must be taken over the orbital energies to allow for the possible partitioning of the atom's electrons among its orbitals. This is done by calculating the **partition function**, Z, for the initial and final atoms.

Z is the weighted sum of the number of ways the atom can arrange its electrons with the same energy. More energetic (less likely) receive less weight from the Boltzmann factor exp(-E/kT).

Let $E_j$ be the energy of the jth energy level and $g_j$ be the degeneracy of that level, then,

$$Z_i = \sum g_j \exp( -[E_j - E_1]/kT )$$

where sum is over all j =1 to ∞.

# Spectral Classification of Stars
## Physical Description

$$Z = \sum g_j \exp( -[E_j-E_1]/kT )$$
where sum is over all $j = 1$ to $\infty$.

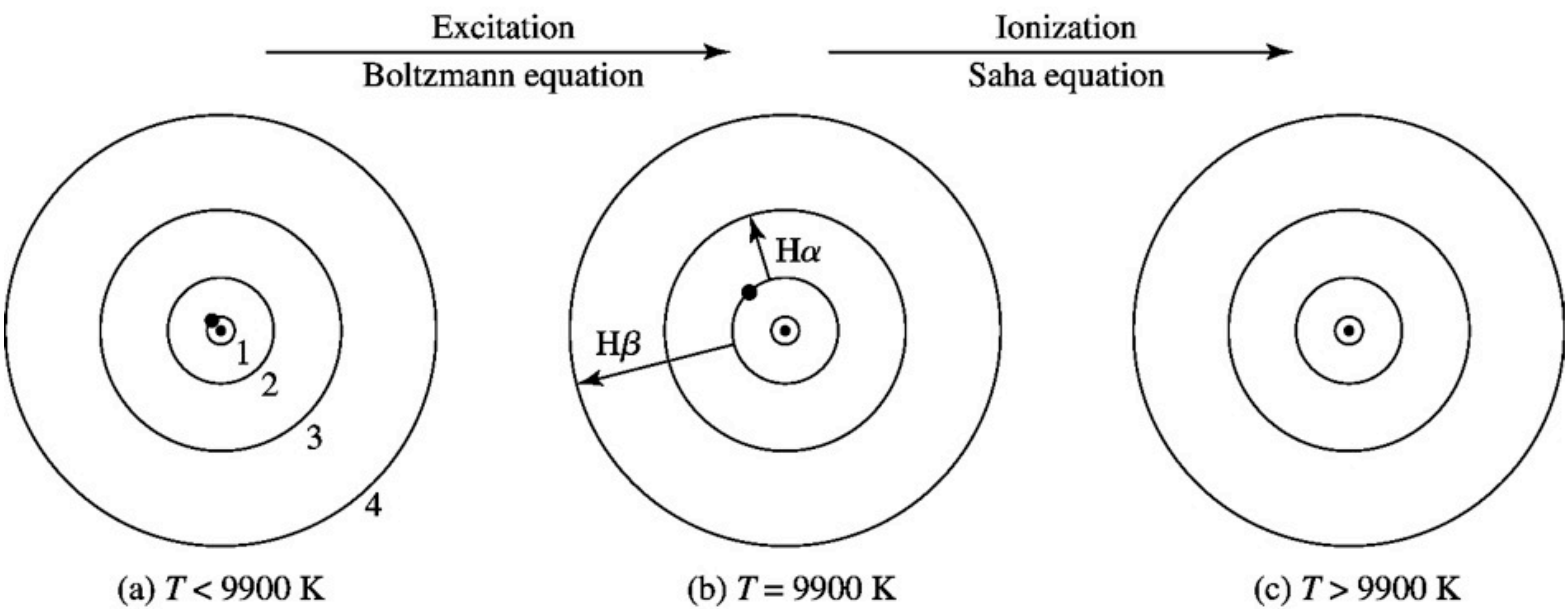
Using the Partition Functions $Z_i$ and $Z_{i+1}$ for the atom in its initial (i) and final (i+1) stages of ionization, the ratio of the # of atoms in stage (i+1) to those in stage (i) is

$$\frac{N_{i+1}}{N_i} = \frac{2Z_{i+1}}{n_e Z_i} \left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} \exp( -\chi_i / kT )$$

This is the **Saha Equation** (named for Megh Nad Saha, 1894-1956, Indian astrophysicist) who worked it out.

$n_e$ is the # of *free* electrons (those *not* bound to atoms). As $n_e$ goes up, the ratio of $N_{i+1}$ to $N_i$ goes down because more unbound electrons are available to combine with ionized atoms.

# Spectral Classification of Stars
## Physical Description



# Spectral Classification of Stars
## Physical Description

Using $P_e = n_e kT$, (Ideal gas law) can rewrite Saha equation as,

$$\frac{N_{i+1}}{N_i} = \frac{2kT\, Z_{i+1}}{P_e\, Z_i} \left( \frac{2\pi m_e kT}{h^2} \right)^{3/2} \exp(-\chi_i / kT)$$

Combine **Saha** and **Boltzmann Equations.**

Consider the ionization of a star's atmosphere composed of pure hydrogen with $P_e = 20$ N m$^{-2}$. Calculate $N_{II}/N_{total} = N_{II} / (N_I + N_{II})$. Consider range of T = 5000 to 25,000 K.

**Calculate partition functions:**

$Z_{II}$ is just that for a single (ionized) proton, $Z_{II}=1$.

For $Z_I$, energy difference of the ground and the first excited states is (-3.40 eV) - (-13.6 eV) = 10.2 eV. This is much greater than kT (=0.43-2.2 eV for range above), so $\exp(-\Delta E/kT) < 0.01 \ll 1$. Therefore, $Z = \sum g_j \exp(-[E_j - E_1]/kT)$ simplifies greatly, and $Z_I \approx g_1 = 2$.

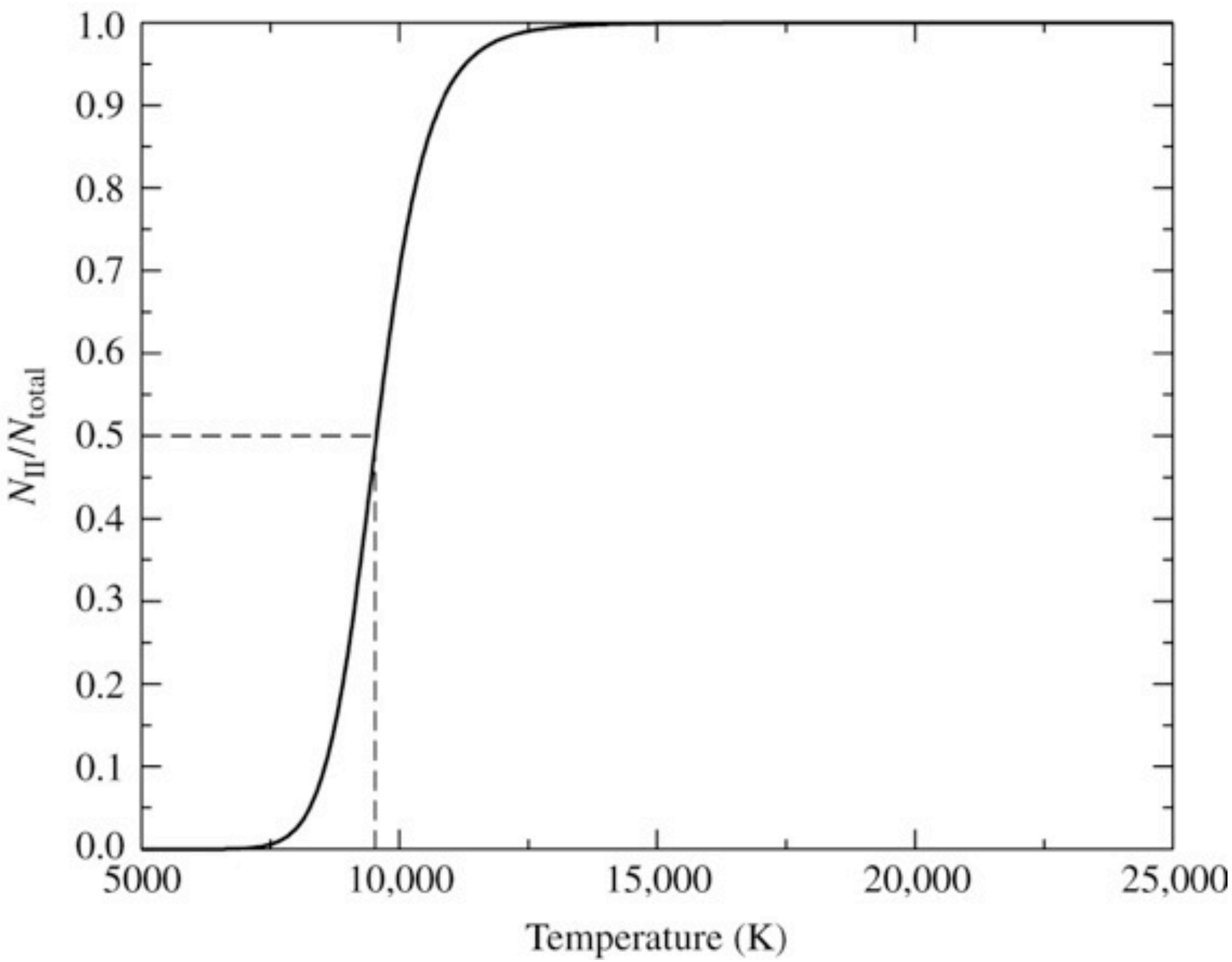
# Spectral Classification of Stars
## Physical Description

$$\frac{N_{II}}{N_I} = \frac{2kT\, Z_{II}}{P_e\, Z_I} \left( \frac{2\pi m_e\, kT}{h^2} \right) \exp( -\chi_I / kT )$$

with $Z_{II}=1$ and $Z_I \approx g_1 = 2$.

$N_{II} /(N_I + N_{II}) =$

$( N_{II}/ N_I) / ( 1 + N_{II} / N_I)$

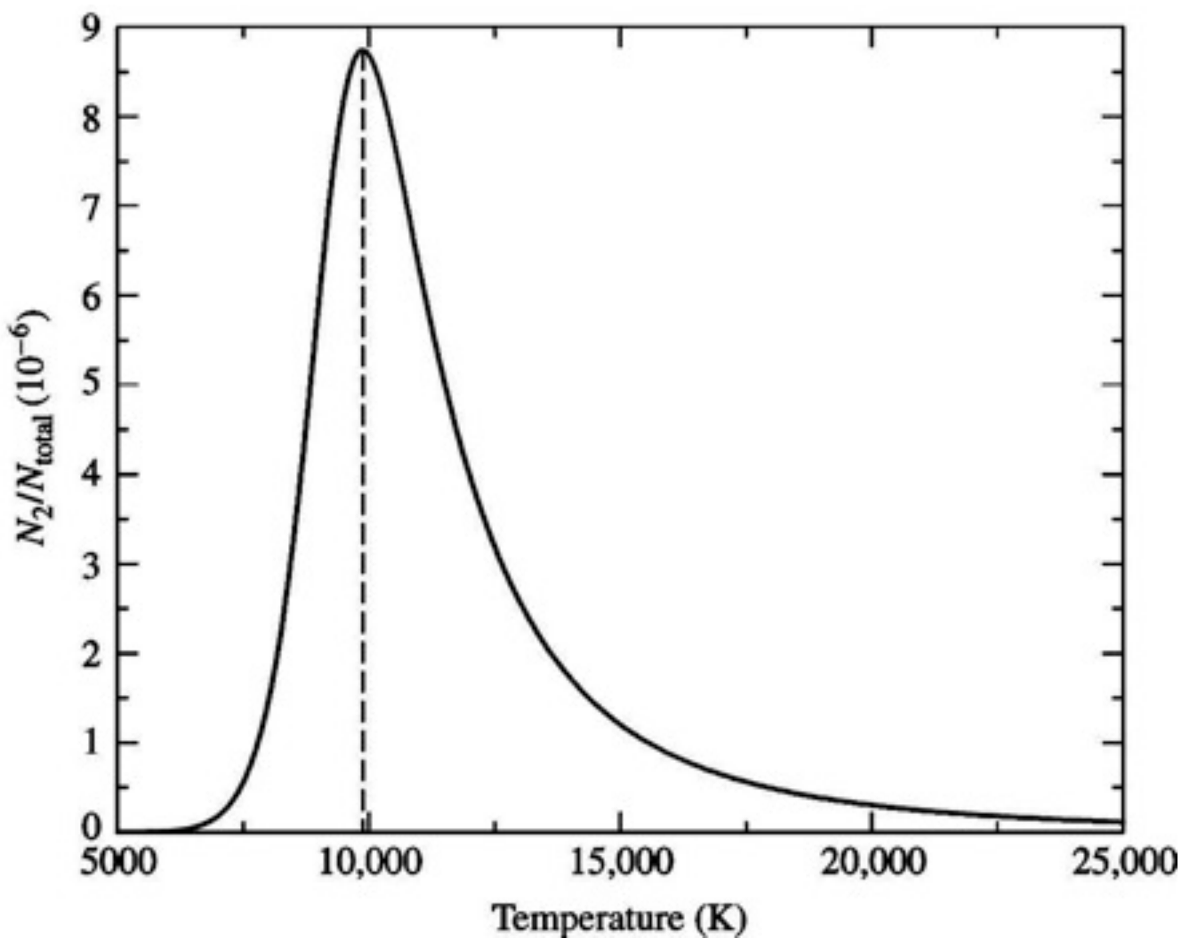


# Spectral Classification of Stars
## Physical Description

Note that $N_2 \neq N_{II}$ !

The strength of the Balmer lines depends on $N_2 / N_{total}$. (Fraction of hydrogen in first excited state to total.) Because *most* neutral hydrogen is in either first excited state or ground state, we can approximate $N_1+N_2 \approx N_I$ and then

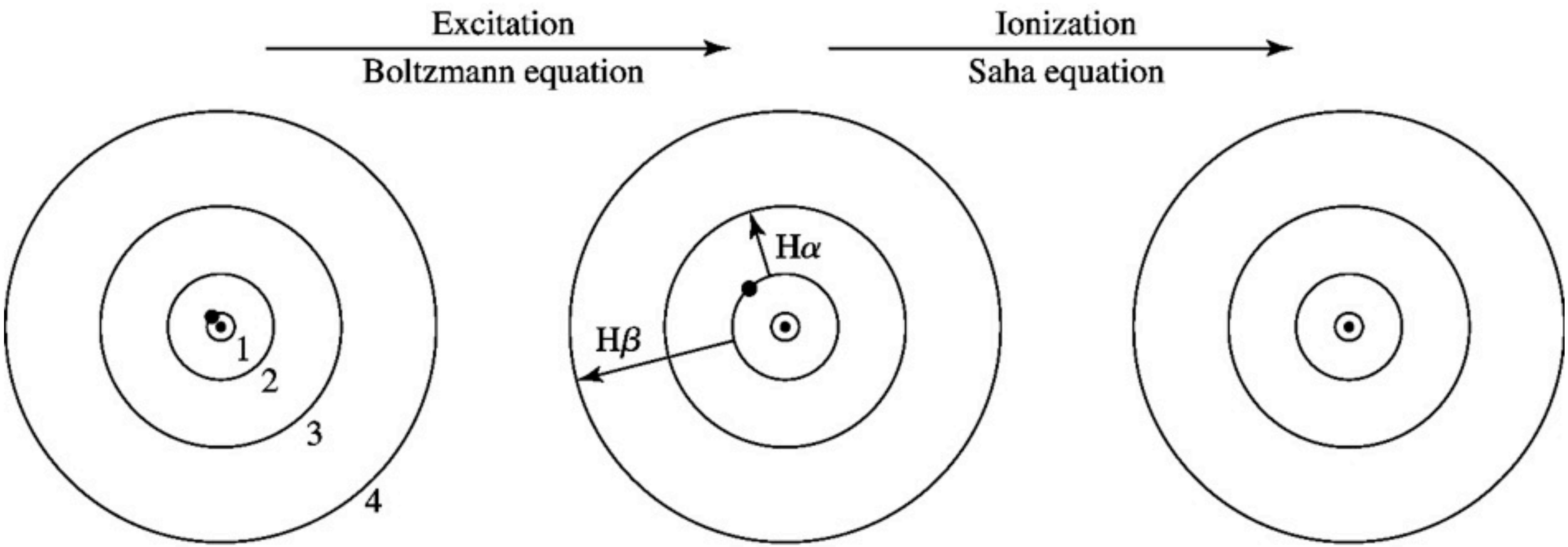
$$\frac{N_2}{N_{total}} = \left(\frac{N_2}{N_1+N_2}\right)\left(\frac{N_I}{N_{tot}}\right) = \left(\frac{N_2/N_1}{1+N_2/N_1}\right)\left(\frac{1}{1+N_{II}/N_I}\right)$$




Hydrogen gas produces most intense Balmer lines at T=9900 K (excellent agreement with observations).

Diminishing strength of the Balmer lines at higher T is due to *ionization* of hydrogen at T > 10,000 K.

# Spectral Classification of Stars
## Physical Description




(a) $T < 9900$ K (b) $T = 9900$ K (c) $T > 9900$ K

Hydrogen when T < 9900 K
Majority of electrons in ground state, n=1

Hydrogen when T > 9900 K
Majority of electrons unbound, ionized hydrogen.

Hydrogen when T = 9900 K
Majority of electrons in first excited, n=2 state, and capable of producing Balmer lines

# Spectral Classification of Stars
## Physical Description

Stars are not composed of pure hydrogen, but nearly all atoms (mostly H, He, and *metals* = anything *not* H or He).

Typically 1 He atom for every 10 H atoms (and even fewer *metals*).

Helium (and metals) provide more electrons, which can recombine with ionized H.

So, it takes *higher* temperatures to achieve same degree of H ionization when He and metals are present.

Abundance is = $\log_{10}(N_{element}/N_H) + 12$.
I.e., Abudance of Oxygen = 8.83, which means:
$8.83 = \log10(N_O/N_H) + 12$
$N_O/N_H = 10^{8.83 - 12} = 0.000676 \approx 1/1480$
There is one Oxygen atom
for every 1480 H atoms !

Most Abundant Elements in the Solar Photosphere.

| Element | Atomic # | Log Relative Abundance |
|---|---|---|
| H | 1 | 12.00 |
| He | 2 | 10.93 ± 0.004 |
| O | 8 | 8.83 ± 0.06 |
| C | 6 | 8.52 ± 0.06 |
| Ne | 10 | 8.08 ± 0.06 |
| N | 7 | 7.92 ± 0.06 |
| Mg | 12 | 7.58 ± 0.05 |
| Si | 14 | 7.55 ± 0.05 |
| Fe | 26 | 7.50 ± 0.05 |
| S | 16 | 7.33 ± 0.11 |
| Al | 13 | 6.47 ± 0.07 |
| Ar | 18 | 6.40 ± 0.06 |
| Ca | 20 | 6.36 ± 0.02 |
| Ng | 11 | 6.33 ± 0.03 |
| Ni | 28 | 6.25 ± 0.04 |

# Spectral Classification of Stars
## Physical Description

Consider a Sun-like star with T=5777 K and 500,000 H atoms for each Calcium (Ca) atom with $P_e$ = 1.5 N m$^{-2}$. Calculate relative strength of Balmer and singly ionized Ca (Ca II).

Hydrogen: Saha equation gives ratio of ionized to neutral atoms.

$$\frac{N_{II}}{N_I} = \frac{2kT\, Z_{i+1}}{P_e\, Z_i}\left(\frac{2\pi m_e\, kT}{h^2}\right)^{3/2} \exp(-\chi_i / kT) = 7.7 \times 10^{-5} = 1 / 13{,}000$$

1 ionized hydrogen ion (H II) for every 13,000 neutral hydrogen atoms

Hydrogen: Boltzmann equation gives ratio of atoms in first excited state to ground state.

$$\frac{N_2}{N_1} = (g_2/g_1) \exp(-[E_2-E_1]/kT) = 5.06 \times 10^{-9} = 1 / 198{,}000{,}000$$

1 hydrogen ion in the first excited state for every 198 *million* hydrogen atoms in the ground state.

# Spectral Classification of Stars
## Physical Description

Calcium: ionization energy $\chi_I$ of CaI is 6.11 eV (roughly half that of Hydrogen. Again, $\chi_I >> kT$ (kT=0.5 eV), so $\exp(-\chi_i / kT)$ is very small ($4.1 \times 10^{-6}$). Complicated derivation, but $Z_{II} = 2.30$ and $Z_I = 1.32$.

$$\text{Calcium:} \quad \frac{N_{II}}{N_I} = \frac{2kT\, Z_{II}}{P_e\, Z_I} \left( \frac{2\pi m_e\, kT}{h^2} \right)^{3/2} \exp(-\chi_I / kT) = 918.$$

Saha Equation gives that 918 singly ionized calcium atoms (Ca II) for every 1 neutral Calcium atoms

Boltzmann equation gives ratio of atoms in first excited state to ground state. Consider Ca II K line (393 nM): $E_2 - E_1 = 3.12$ eV and $g_1 = 2$ and $g_2 = 4$.

$$\text{Calcium:} \quad \frac{N_2}{N_1} = (g_2/g_1) \exp(-[E_2 - E_1]/kT) = 3.79 \times 10^{-3} = 1 / 264$$

Out of 264 singly ionized calcium ions (Ca II), all but one are in the ground state (the 263 others are capable of producing the Ca II K line).

# Spectral Classification of Stars
## Physical Description

Combining Saha and Boltzmann equations for Calcium:

$$\left(\frac{N_I}{N_{total}}\right)_{CaII} \approx \left(\frac{N_I}{N_I+N_2}\right)_{CaII} \left(\frac{N_{II}}{N_{tot}}\right)_{Ca} = \left(\frac{1}{1+N_2/N_I}\right)_{CaII} \left(\frac{N_{II}/N_I}{1+N_{II}/N_I}\right)_{Ca}$$

$$= (1 / (1+3.79\times10^{-3})) \times (918 / (1+918) = 0.995$$

Nearly *all* the calcium atoms can produce Ca II K emission.

There are 500,000 H atoms for every Ca atom. *But* only a small fraction ($5 \times 10^{-9}$) of these H atoms are neutral and in the first excited state (most in ground state).

$500{,}000 \times (5 \times 10^{-9}) = 1 / 400.$

Approximately 400 times more Ca II ions to produce the Ca II lines than there are neutral H atoms in the first excited state. This is why the Sun (a G5 star) has strong Ca II and relatively weak Balmer lines.

# Spectral Classification of Stars
## Physical Description

In 1925, Cecilia Payne (1900-1979) calculate the relative abundances of 18 elements in stellar atmospheres (one of the most brilliant PhD theses ever in astronomy).