Stellar Abundances

I. Basic principles of stellar nucleosynthesis
II. Basic ingredients
III. Deciphering abundances
IV. A case study: LiBeB

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Basic ingredients

- Stellar atmospheres and line formation
- Characteristics of star → stellar parameters
- From lines to abundances
- Lines (atomic and/or molecular)
- Model atmospheres
- Available tools

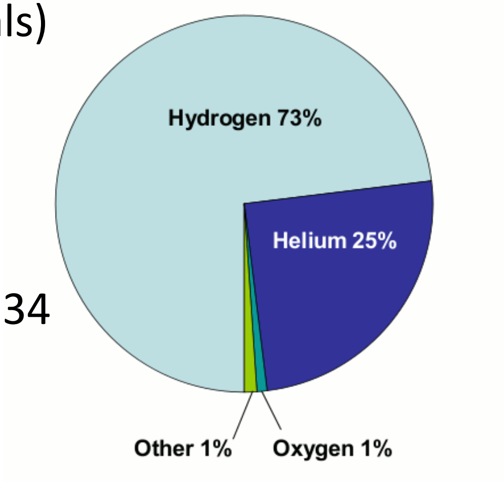
Abundance Scales (and nomenclature)

Mass fractions

X=H; Y=He, Z=all other elements (=metals)

X+Y+Z = 1

EX: cf. 1st lecture →

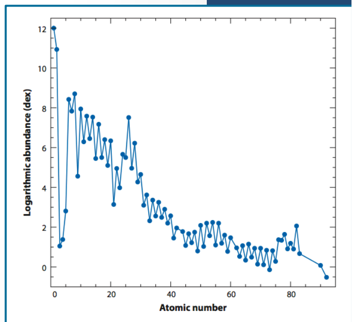


X_{sun}= 0.7381, Y_{sun}= 0.2485, Z_{sun}= 0.0134
(Asplund et al. 2009)

The 12 scale

$\log \varepsilon(X) = \log (n_X / n_H) + 12$ ($\log \varepsilon(H) \equiv 12$)

EX: $\log \varepsilon(O)_{sun} \approx 8.7$ dex (O is ≈2000× less abundant than H_{sun})



The [] scale

$[X/H] = \log (n_X / n_H)_* - \log (n_X / n_H)_{sun}$

EX: [Fe/H] = –2 → star has 1/100 less iron than the Sun

[Fe/H] = 0.5 → star has 3.16× more iron than the Sun

Present-day solar photosphere elemental abundances

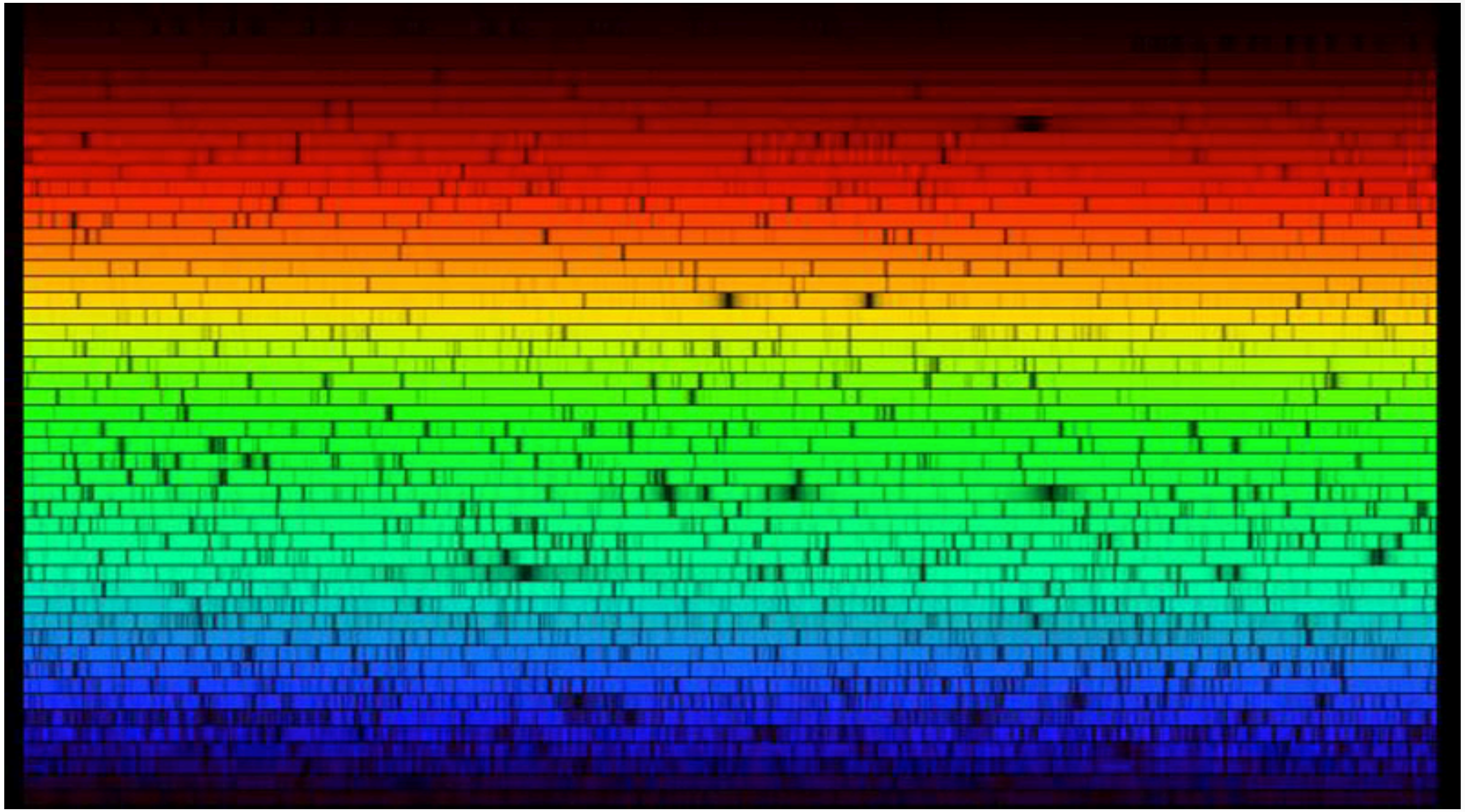
Lodders et al. 2009

Asplund et al 2009

Table 1 Element abundances in the present-day solar photosphere. Also given are the corresponding values for CI carbonaceous chondrites (Lodders, Palme & Gail 2009). Indirect photospheric estimates have been used for the noble gases (Section 3.9)

Z	Element	Photosphere	Meteorites	Z	Element	Photosphere	Meteorites
1	H	12.00	8.22 ± 0.04	44	Ru	1.75 ± 0.08	1.76 ± 0.03
2	He	[10.93 ± 0.01]	1.29	45	Rh	0.91 ± 0.10	1.06 ± 0.04
3	Li	1.05 ± 0.10	3.26 ± 0.05	46	Pd	1.57 ± 0.10	1.65 ± 0.02
4	Be	1.38 ± 0.09	1.30 ± 0.03	47	Ag	0.94 ± 0.10	1.20 ± 0.02
5	B	2.70 ± 0.20	2.79 ± 0.04	48	Cd		1.71 ± 0.03
6	C	8.43 ± 0.05	7.39 ± 0.04	49	In	0.80 ± 0.20	0.76 ± 0.03
7	N	7.83 ± 0.05	6.26 ± 0.06	50	Sn	2.04 ± 0.10	2.07 ± 0.06
8	O	8.69 ± 0.05	8.40 ± 0.04	51	Sb		1.01 ± 0.06
9	F	4.56 ± 0.30	4.42 ± 0.06	52	Te		2.18 ± 0.03
10	Ne	[7.93 ± 0.10]	−1.12	53	I		1.55 ± 0.08
11	Na	6.24 ± 0.04	6.27 ± 0.02	54	Xe	[2.24 ± 0.06]	−1.95
12	Mg	7.60 ± 0.04	7.53 ± 0.01	55	Cs		1.08 ± 0.02
13	Al	6.45 ± 0.03	6.43 ± 0.01	56	Ba	2.18 ± 0.09	2.18 ± 0.03
14	Si	7.51 ± 0.03	7.51 ± 0.01	57	La	1.10 ± 0.04	1.17 ± 0.02
15	P	5.41 ± 0.03	5.43 ± 0.04	58	Ce	1.58 ± 0.04	1.58 ± 0.02
16	S	7.12 ± 0.03	7.15 ± 0.02	59	Pr	0.72 ± 0.04	0.76 ± 0.03
17	Cl	5.50 ± 0.30	5.23 ± 0.06	60	Nd	1.42 ± 0.04	1.45 ± 0.02
18	Ar	[6.40 ± 0.13]	−0.50	62	Sm	0.96 ± 0.04	0.94 ± 0.02
19	K	5.03 ± 0.09	5.08 ± 0.02	63	Eu	0.52 ± 0.04	0.51 ± 0.02
20	Ca	6.34 ± 0.04	6.29 ± 0.02	64	Gd	1.07 ± 0.04	1.05 ± 0.02
21	Sc	3.15 ± 0.04	3.05 ± 0.02	65	Tb	0.30 ± 0.10	0.32 ± 0.03
22	Ti	4.95 ± 0.05	4.91 ± 0.03	66	Dy	1.10 ± 0.04	1.13 ± 0.02
23	V	3.93 ± 0.08	3.96 ± 0.02	67	Ho	0.48 ± 0.11	0.47 ± 0.03
24	Cr	5.64 ± 0.04	5.64 ± 0.01	68	Er	0.92 ± 0.05	0.92 ± 0.02
25	Mn	5.43 ± 0.04	5.48 ± 0.01	69	Tm	0.10 ± 0.04	0.12 ± 0.03
26	Fe	7.50 ± 0.04	7.45 ± 0.01	70	Yb	0.84 ± 0.11	0.92 ± 0.02
27	Co	4.99 ± 0.07	4.87 ± 0.01	71	Lu	0.10 ± 0.09	0.09 ± 0.02
28	Ni	6.22 ± 0.04	6.20 ± 0.01	72	Hf	0.85 ± 0.04	0.71 ± 0.02
29	Cu	4.19 ± 0.04	4.25 ± 0.04	73	Ta		−0.12 ± 0.04
30	Zn	4.56 ± 0.05	4.63 ± 0.04	74	W	0.85 ± 0.12	0.65 ± 0.04
31	Ga	3.04 ± 0.09	3.08 ± 0.02	75	Re		0.26 ± 0.04
32	Ge	3.65 ± 0.10	3.58 ± 0.04	76	Os	1.40 ± 0.08	1.35 ± 0.03
33	As		2.30 ± 0.04	77	Ir	1.38 ± 0.07	1.32 ± 0.02
34	Se		3.34 ± 0.03	78	Pt		1.62 ± 0.03
35	Br		2.54 ± 0.06	79	Au	0.92 ± 0.10	0.80 ± 0.04
36	Kr	[3.25 ± 0.06]	−2.27	80	Hg		1.17 ± 0.08
37	Rb	2.52 ± 0.10	2.36 ± 0.03	81	Tl	0.90 ± 0.20	0.77 ± 0.03
38	Sr	2.87 ± 0.07	2.88 ± 0.03	82	Pb	1.75 ± 0.10	2.04 ± 0.03
39	Y	2.21 ± 0.05	2.17 ± 0.04	83	Bi		0.65 ± 0.04
40	Zr	2.58 ± 0.04	2.53 ± 0.04	90	Th	0.02 ± 0.10	0.06 ± 0.03
41	Nb	1.46 ± 0.04	1.41 ± 0.04	92	U		−0.54 ± 0.03
42	Mo	1.88 ± 0.08	1.94 ± 0.04				

How do you extract elemental abundances from these lines?



- Star ID (spectral type or photometric classification)
- Atmospheric properties → line formation
  - Effective temperature
  - Surface gravity
  - “Metallicity”

- Stellar atmospheres and line formation
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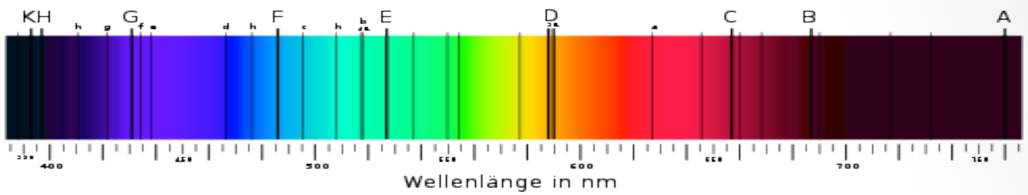
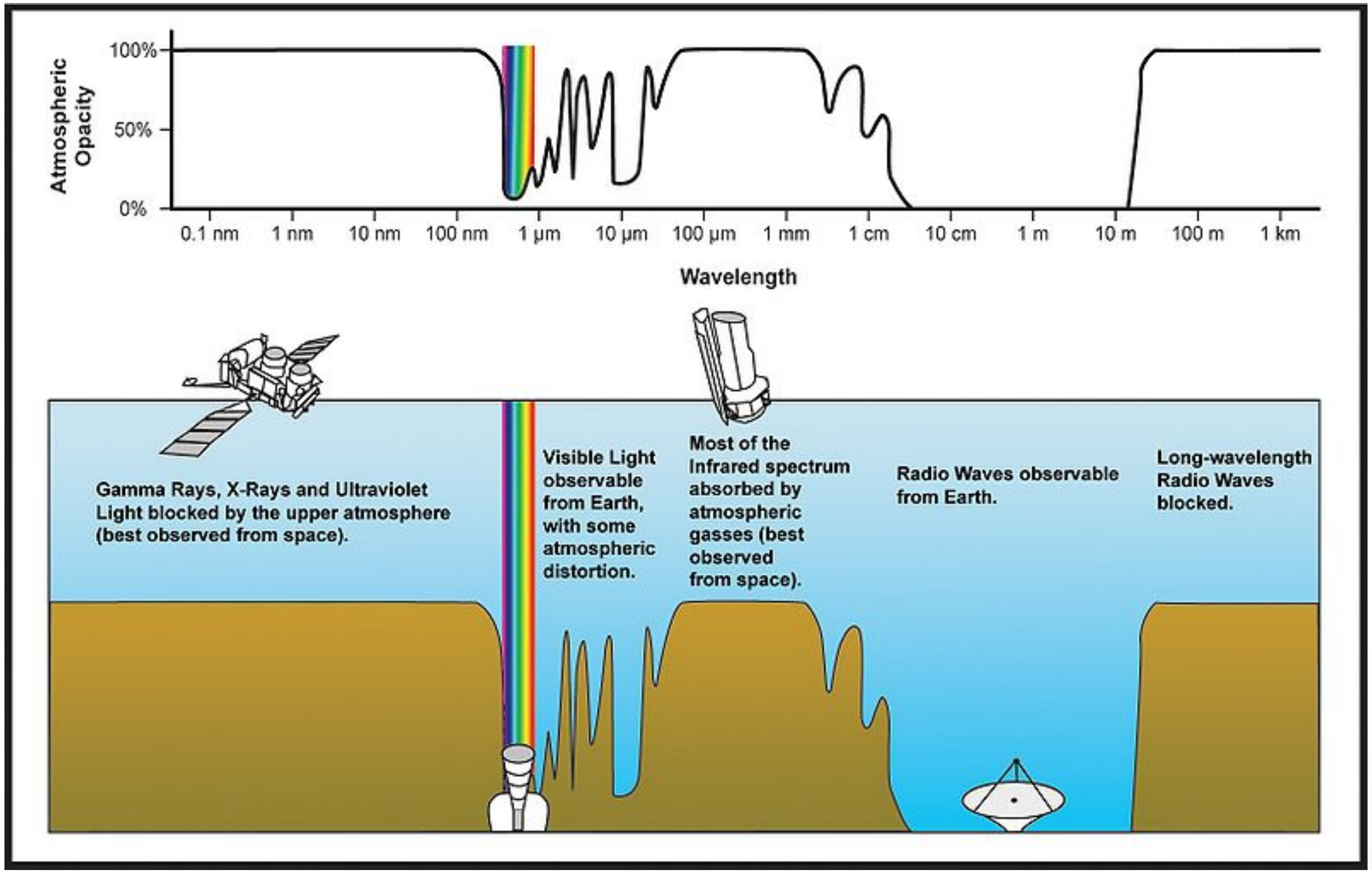
What information does the observed flux carry?

- Absorption-line spectrum

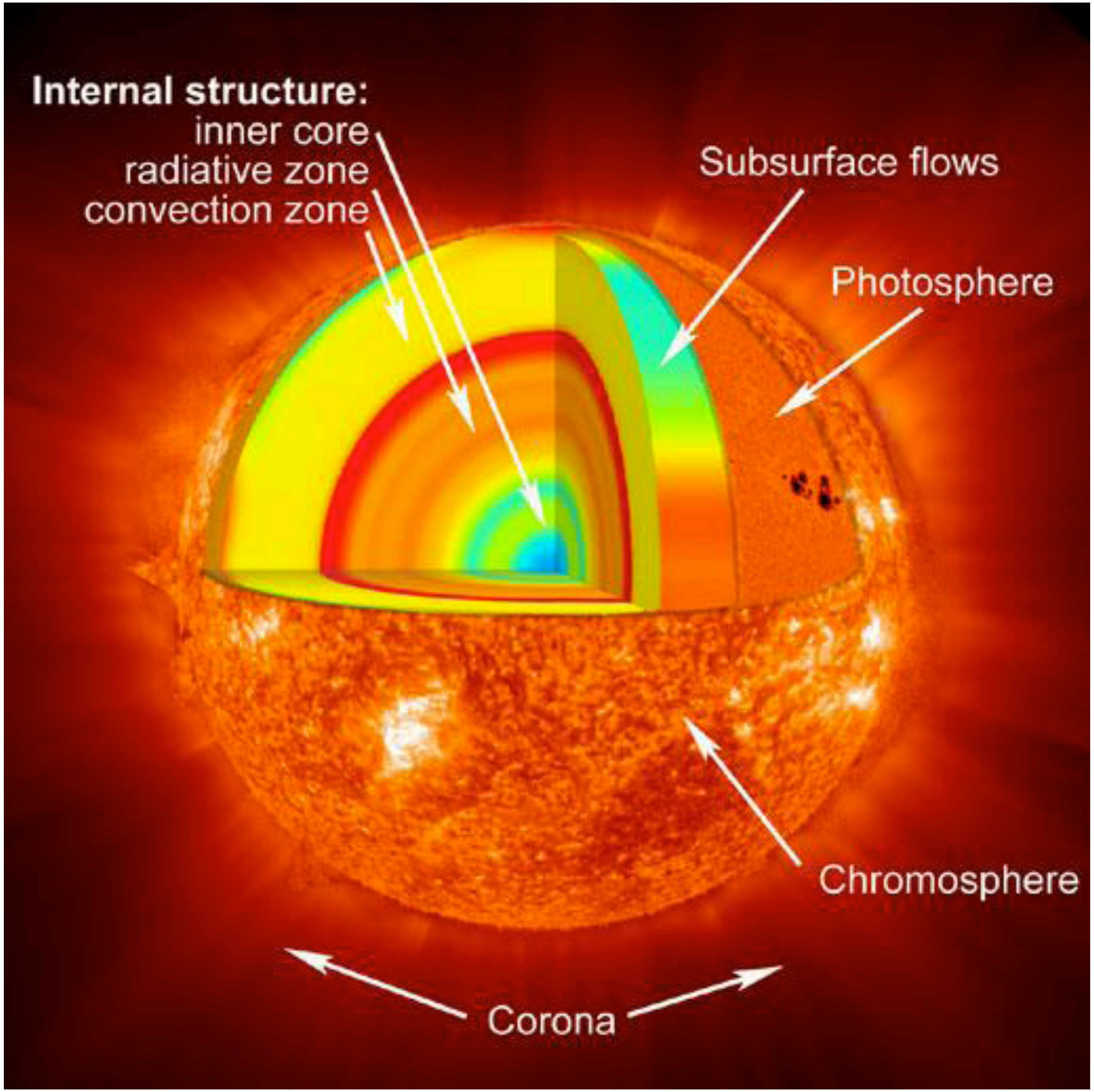


- Lines occur because of atomic absorption → Estimation of elements in the photosphere

- Emergent flux carries information about physical conditions in photosphere



Stellar photosphere: a definition

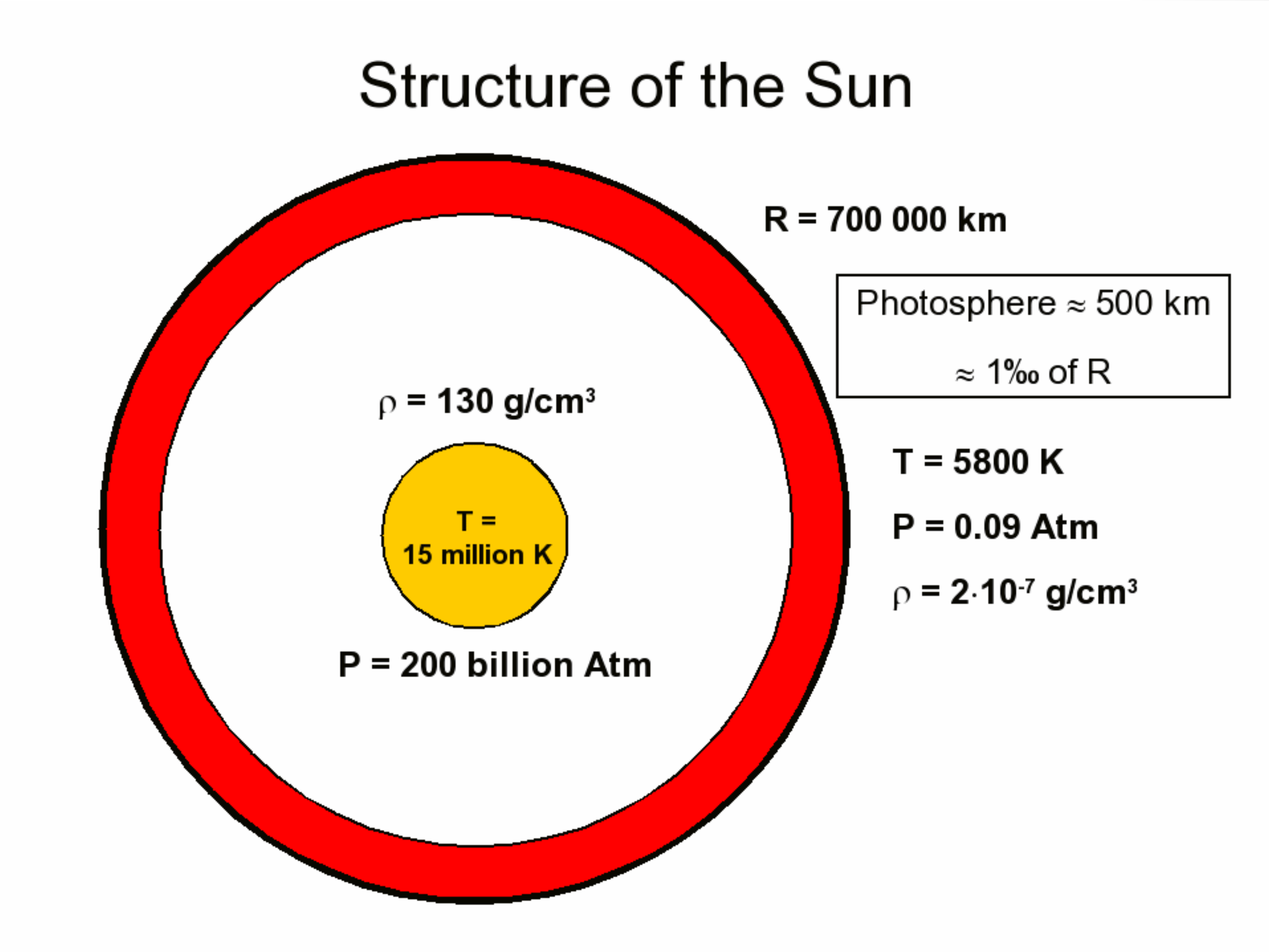


Transition from interior to ISM

Layer from which we receive photons

Source: NASA

Structure of the Sun

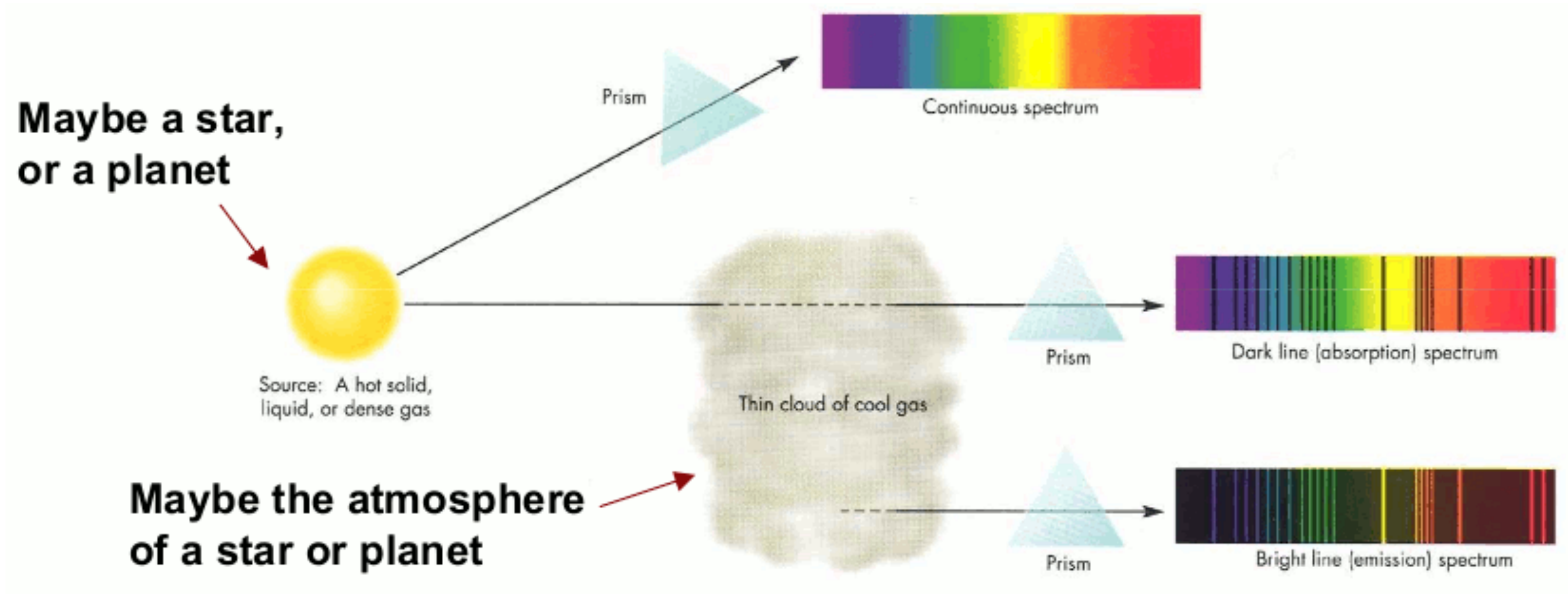


Source: Heiter

Stellar absorption line formation

A cool, thin gas seen in front of a hot source produces absorption lines: in the continuum region, τ is low and we see primarily the background source. At the wavelengths of spectral lines, τ is large and we see the intensity characteristic of the temperature of the cool gas. Since this is lower than central source, these appear as absorption features.

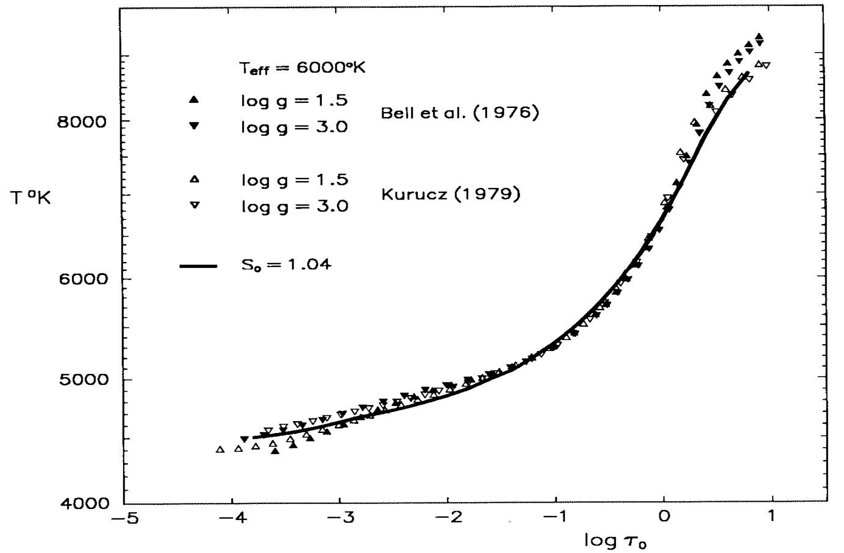
$\tau_\nu = \int_0^L \kappa_\nu \rho dx$



Spectral line formation & profile

The formation of absorption lines can be qualitatively understood by studying how S_ν changes with depth.

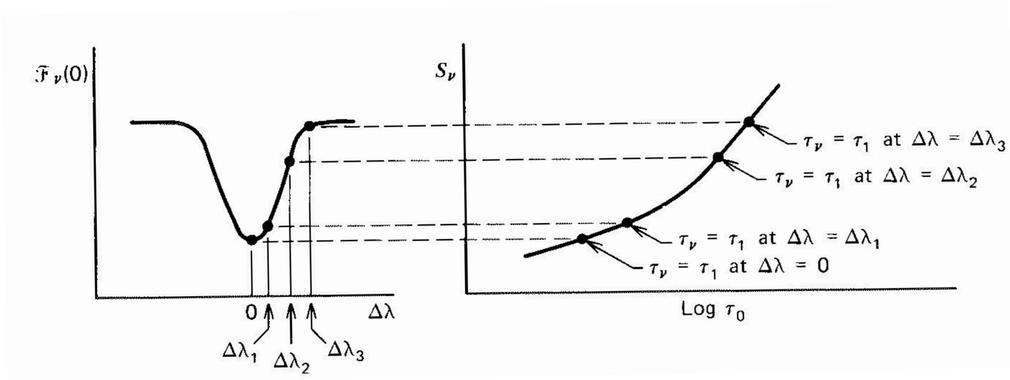
$$W_\lambda \propto d(\ln S_\nu)/d\tau_\nu$$



1. Atmospheric structure

2. Absorption lines : negative T gradient

→ source function ↓ outwards



$$S_\nu^l = \frac{j_\nu}{k_\nu} = \frac{2h\nu^3}{c^2}\frac{1}{\frac{g_2 N_1}{g_1 N_2} - 1}$$

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu$$

3. Transfer equation: solution provides detailed line profile

$$S_\nu = \frac{j_\nu^l + j_\nu^c}{k_\nu^l + k_\nu^c}$$

Source: Gray

Ionization state & energy level

What we want:

1. Enough atoms in the right ionization state (LTE assumption)

Saha's equation

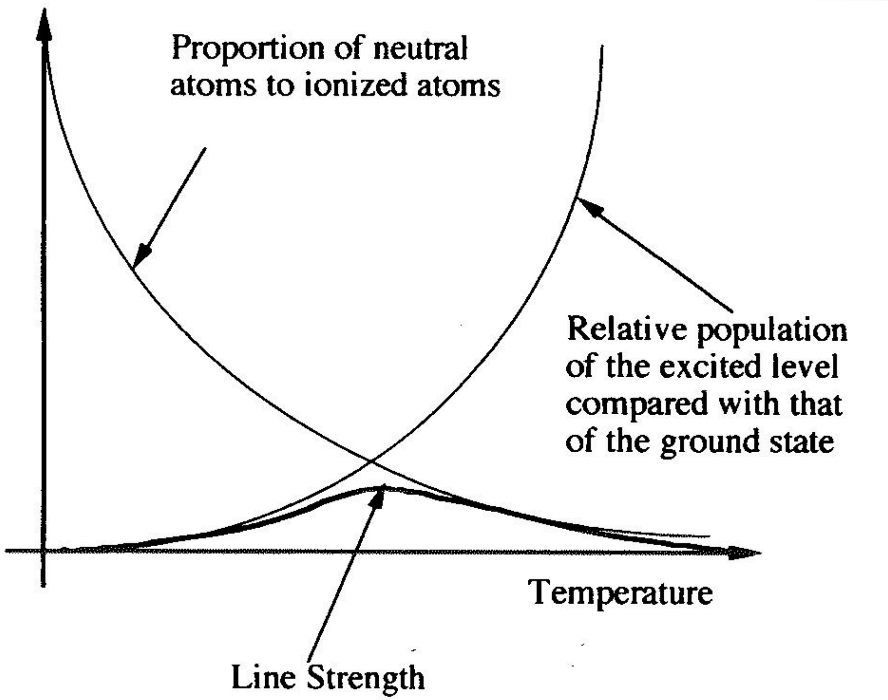
$$\frac{n_1}{n_0} P_e = \frac{(2\pi m_e)^{3/2}(kT)^{5/2}}{h^3} \frac{2u_1(T)}{u_0(T)} e^{-I/kT}$$

2. Enough atoms or ions excited in the right energy level (LTE assumption)

Boltzmann's statistics

$$\frac{N_b}{N_a} = \left(\frac{g_b}{g_a}\right)\left(e^{-(E_b-E_a)/kT}\right)$$

Combination of both →



Line broadenings

3 main components

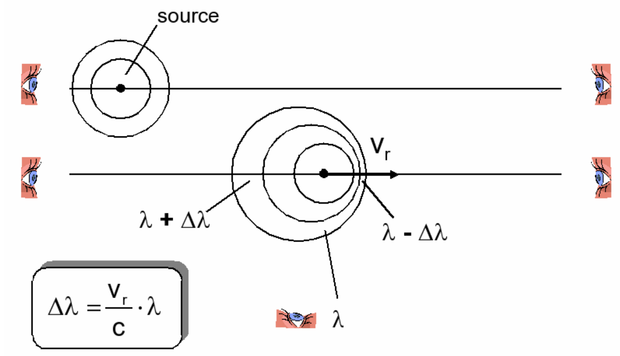
Natural width (Lorentzian profile, very narrow)

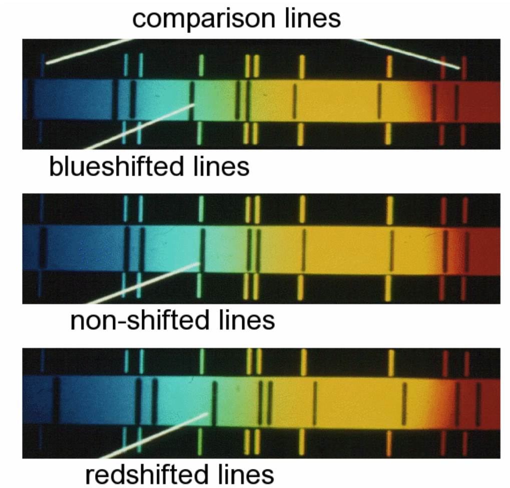
due to Heisenberg uncertainty principle $\Delta E \cdot \Delta\tau = h/2\pi$

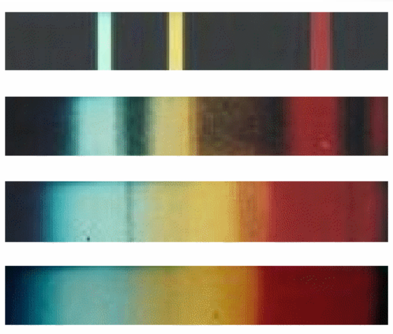
Thermal (Doppler) width (Gaussian, Maxwell-Boltzmann distribution)

thermal motions of the atoms, randomly distributed shifts

→ centre





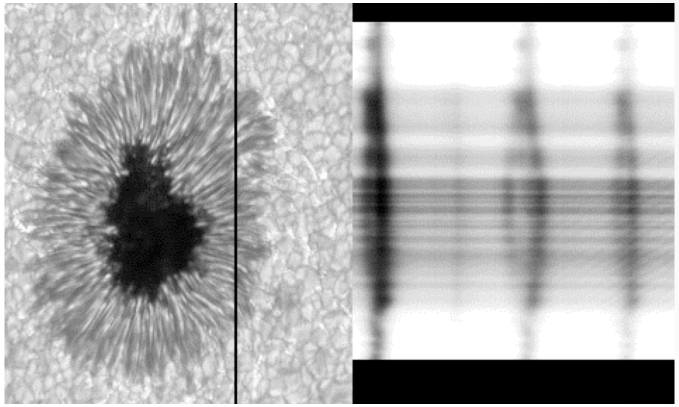


Pressure (collisional) width (Lorentzian profile)

collisions between particles (energy levels change)

→ wings

More broadenings, splits and shifts



Zeeman
In strong magnetic fields atoms can align in quantum ways causing slight separations in the energies of atoms in the same excitation levels.

This "splits" the lines into multiple components. The stronger the field, the greater the splitting.

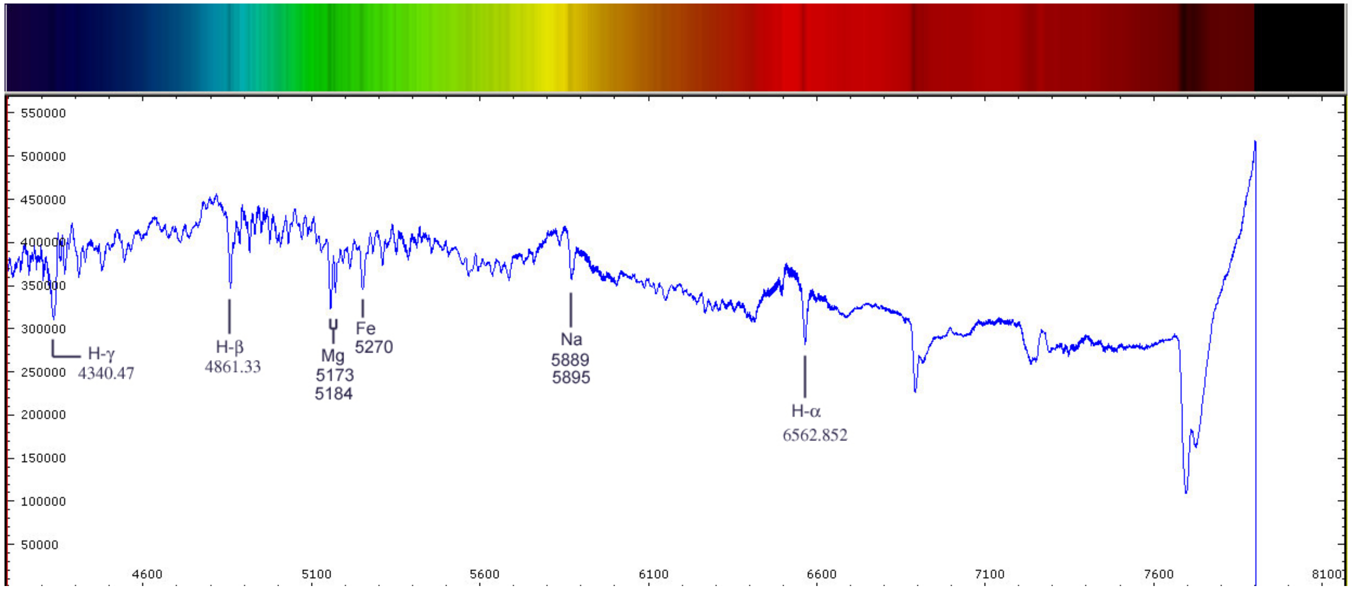
Hyperfine
Interaction between the nuclear magnetic dipole with the magnetic field of the electron can perturb the energy levels of an atom

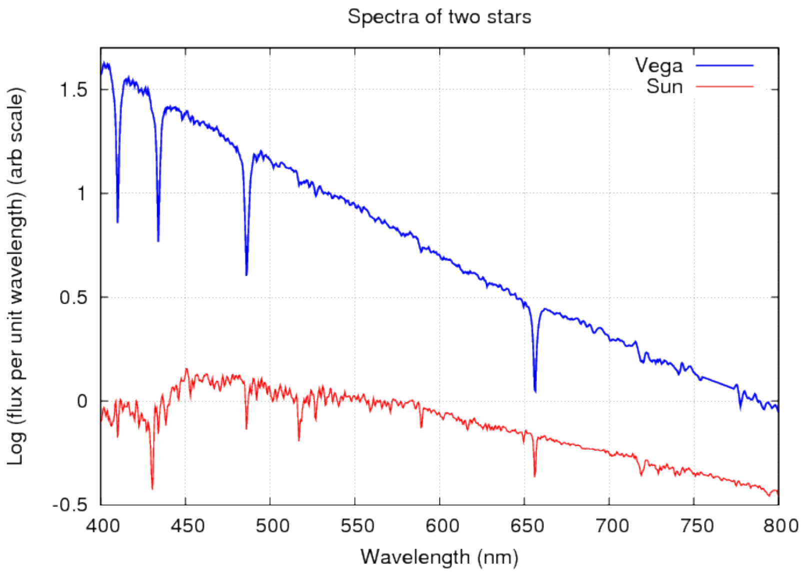
Energy level dependent → different for each line

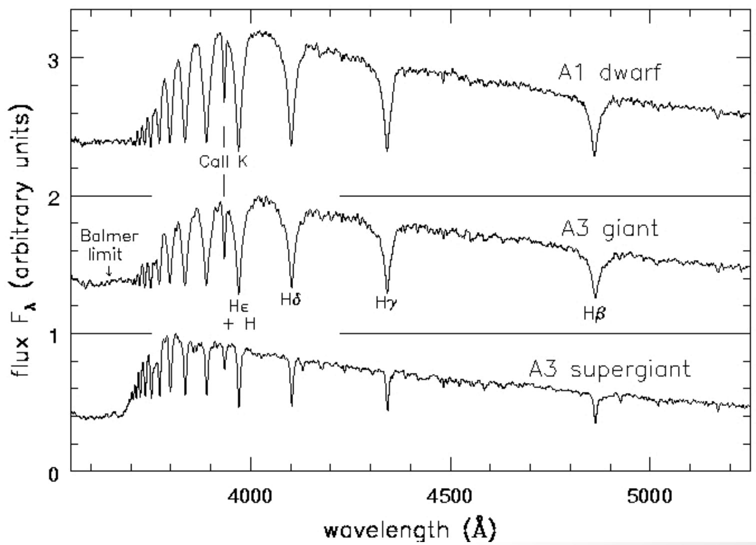
Stark
Perturbation by electric fields

Some examples: Hydrogen lines

Balmer transitions (from the n=2 excited level): strongest in A stars because that is where N (HI, n=2) is highest



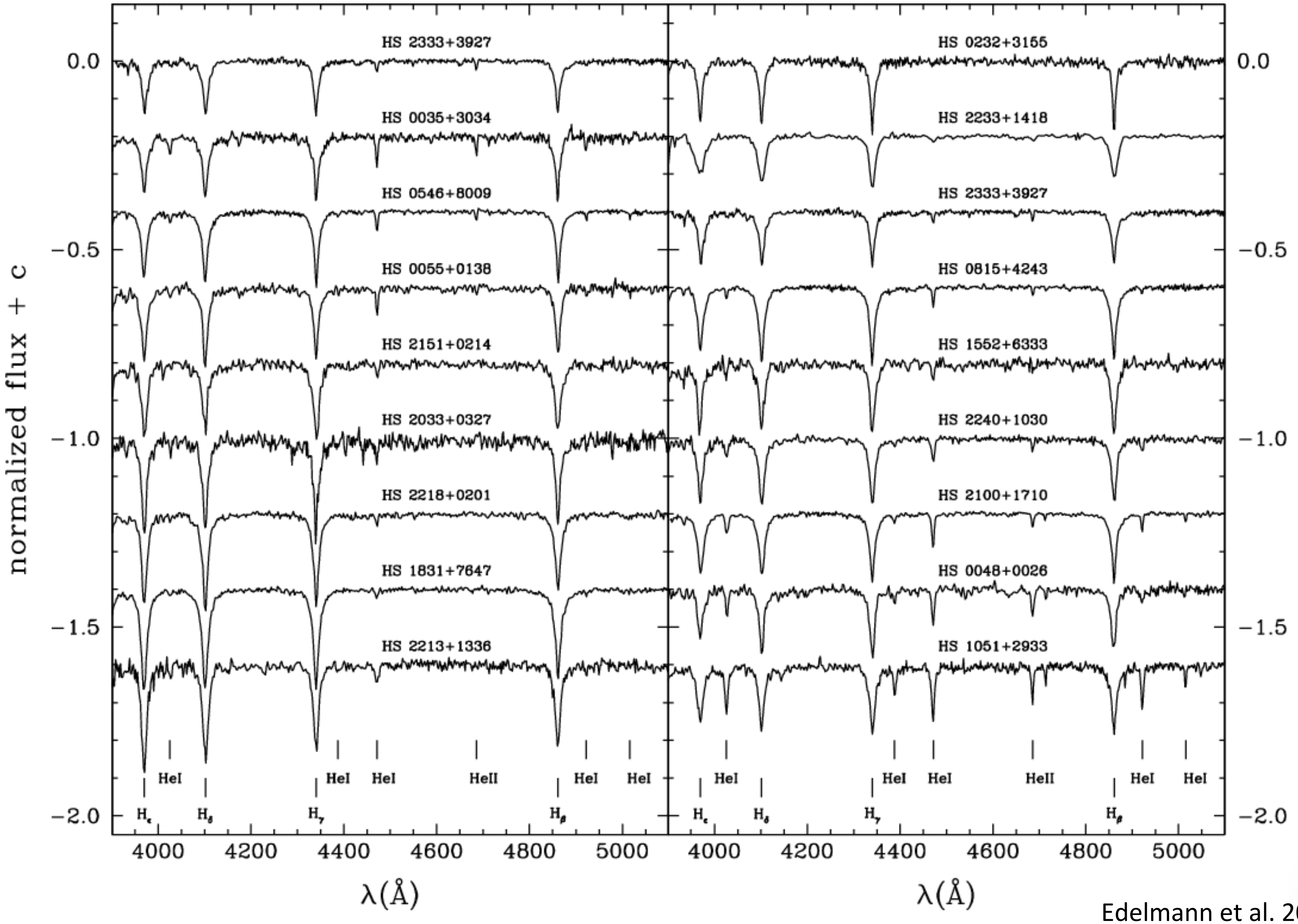




Examples: Helium lines

Second most abundant element

Detectable only in very hot stars, O- and B-types (even HeII in hottest O-stars)



Edelmann et al. 2003, A&A, 400, 393

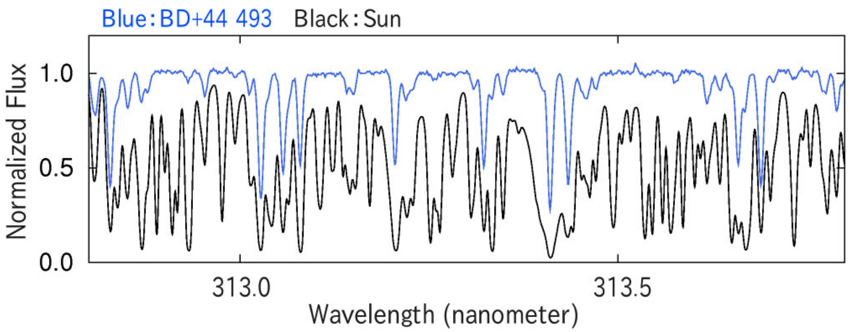
Examples: metal lines

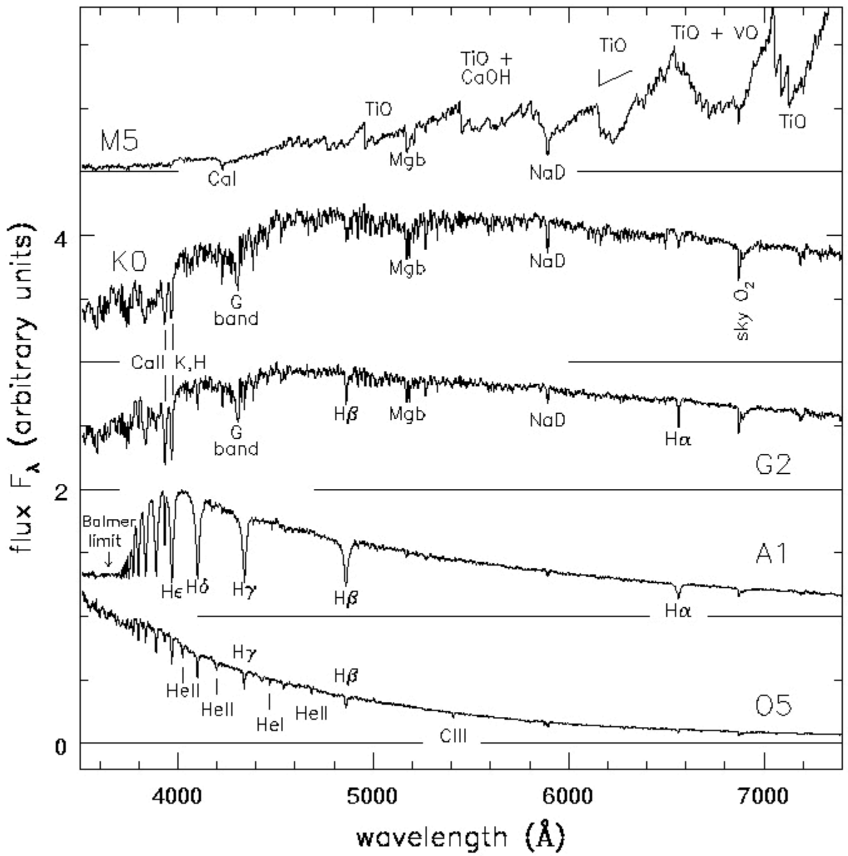
Line strength depends on level population of the atoms.

Strongest when temperature is low (lower ionization stages are populated)

Lines become stronger as T decreases

Dominate in F, G, K stars





Source: Subaru Press Release, Ito et al. 2009, ApJL, 698,37

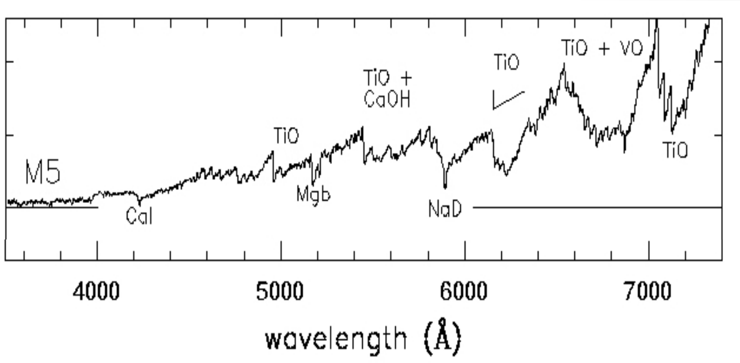
Source: Tolstoy

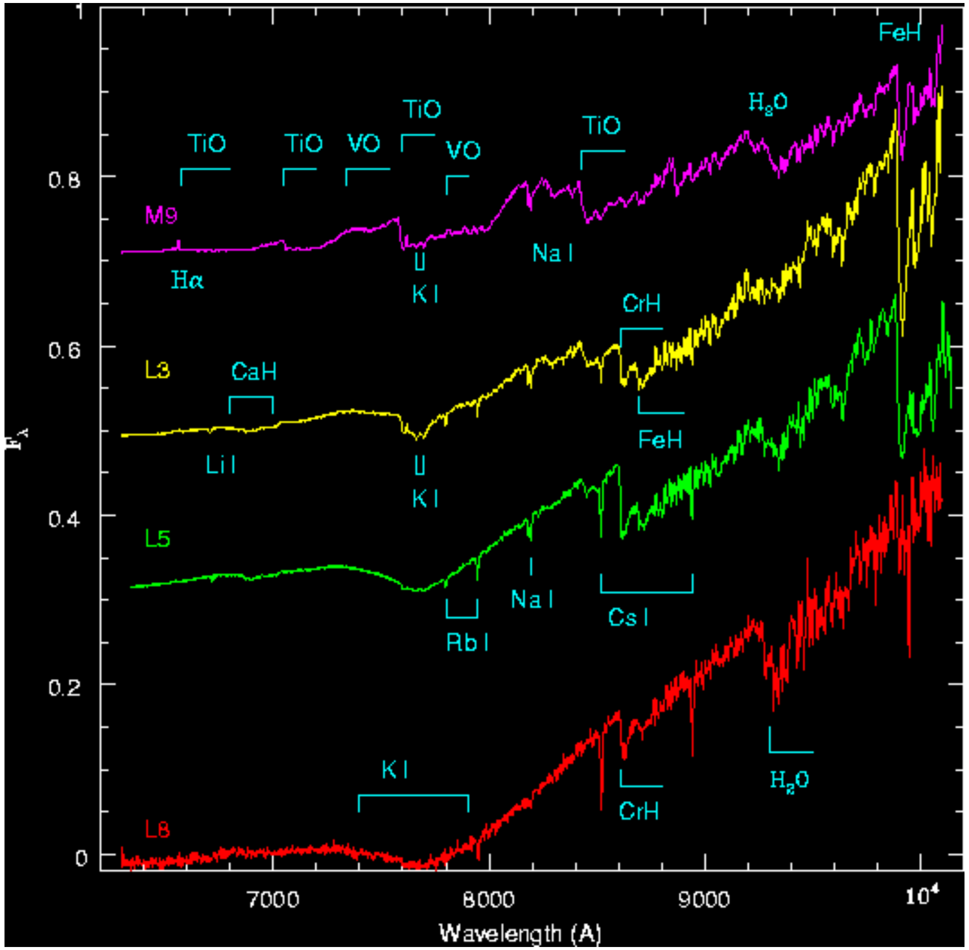
Examples: molecular bands

Form in very cool stars (M-, L-, T-types)

Can vibrate and rotate, besides having energy levels

Flux significantly reduced in bands





Electron transitions:
Visible+UV lines

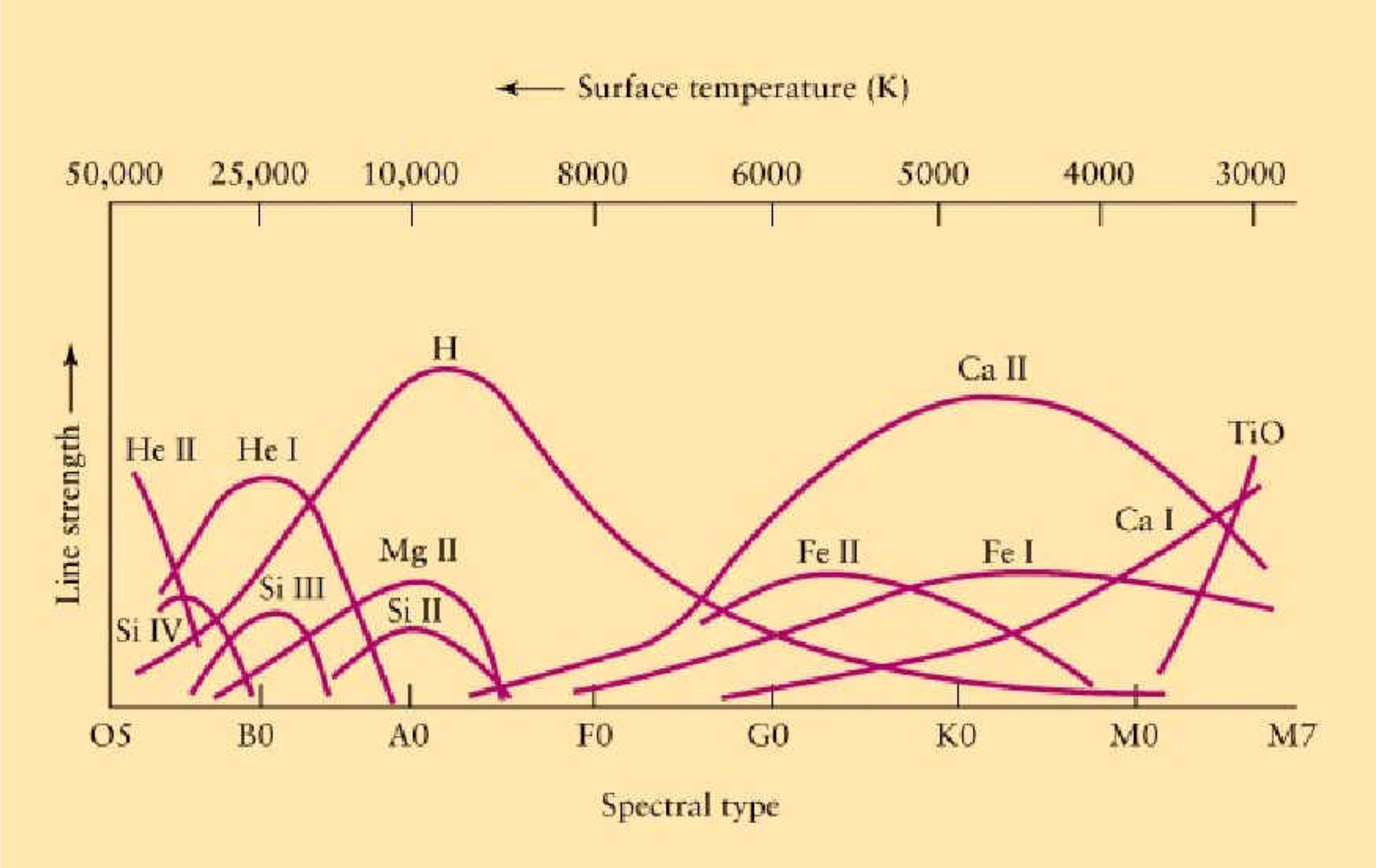
Vibrational transitions:
Infrared lines

Rotational transitions:
Radio-wave lines

M-stars: TiO
L- / T-stars: CO, H_2O and CH^4

Source: Kirckpatrick et al, 1999, 2000

Examples: dominant features



- Stellar atmospheres and line formation
- Characteristics of star → stellar parameters
- From lines to abundances
- Lines (atomic and/or molecular)
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What are the stellar parameters?

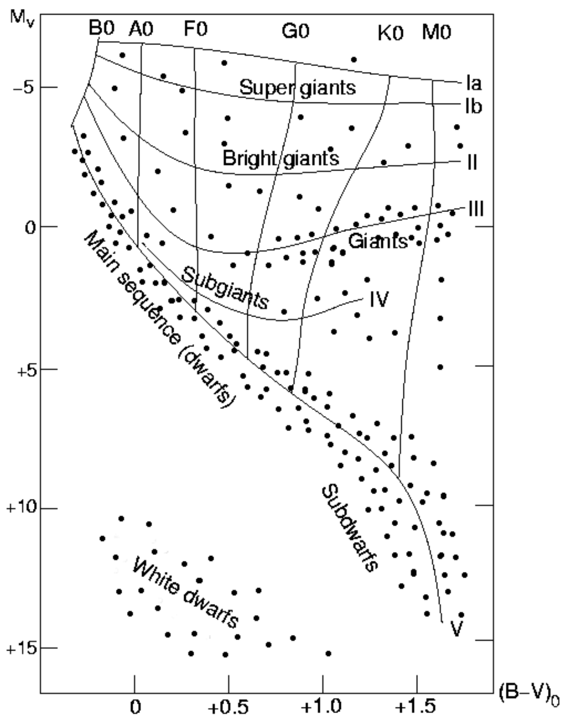
Characteristics of a star:

1. *M*, L, *X*, *Y*, *Z*, *R*, v_{rot}, *t*, … *i.e.*, the stellar-structure view
2. Fν, T_{eff}, log *g*, [X/H], v_{rot}sin *i*, log (G *M*/ *R*2) … i.e., the stellar-atmosphere view

1 → more fundamental
2 → more empirical

Sun
M = 2 x10^{33}g = M_{sun}
R = 7 x10^{10}cm = R_{sun}
L = 4 x10^{33}erg/s = L_{sun}

R_{phot} ~ 200 km < $10^{-3}R_{sun}$
N_{phot} ~ 1015cm^{-3}
T_{eff} ~ 5780K



O star
M ~ 50 M_{sun}
R ~ 20 R_{sun}
L ~ 10^6 L_{sun} (α M^3)

R_{phot} ~ 0.1 R_{sun}
N_{phot} ~ 1014cm^{-3}
T_{eff} ~ 40000 K

Source: Korn

Fundamental stellar parameters: how to

T_{eff} via $\mathcal{F}_{Bol}$ and Θ

For Θ, one uses interferometry and model-atmosphere theory (limb darkening)

Log *g* Newton's law, needs *M* and *R*

One needs π (parallax) and Θ
Gaia is the key π-mission (soon to be launched)

M Eclipsing binaries (very limited)

Must be inferred from stellar evolution

[*m*/H] via meteorites for the Sun

Caveat: lack of important *volatile* elements like CNO and noble gases

Most relevant empirical stellar parameters: how to

Effective temperature T_{eff} = T_{BB} with same L and R as the real star

$L = 4\pi R2\sigma T^4_{eff}$

Surface gravity log *g* usually expressed in cgs units and as $\log_{10}$

$g = GM/R^2$

Metallicity Z or [Fe/H] $Z = \dfrac{mass\ (elements\ heavier\ than\ He)}{total\ mass\ (unit\ volume)} \approx 0.018$ Sun

$[Fe/H] = \log [N(Fe)/N(H)]_* - \log [N(Fe)/N(H)]_{Sun}$

$[\alpha/Fe] = \log [N(\alpha)/N(Fe)]_* - \log [N(\alpha)/N(Fe)]_{Sun}$

where α is an « α-element », i.e. with a nucleus made of an integer number of α-particles

How to determine T_{eff}

<u>Multi-colour photometry</u>

calibrated with stars having fundamentally determined T_{eff}

– Johnson's UBV
– Strömgren's uvby
– Geneva UBV B1 B2 V1 G

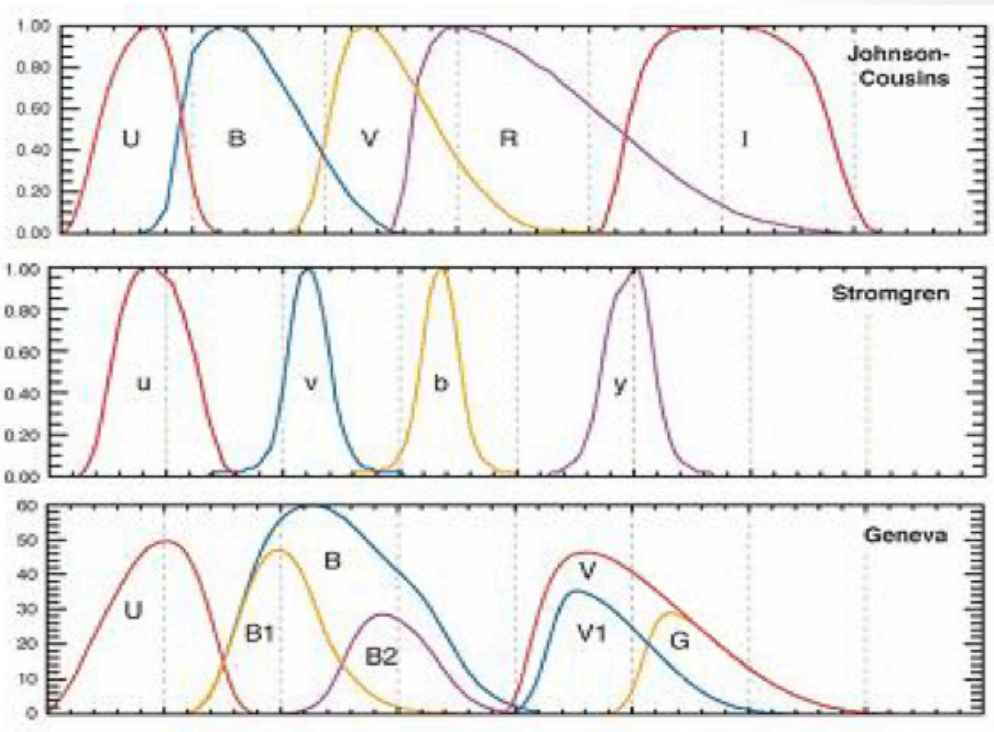
<u>Spectroscopy</u>

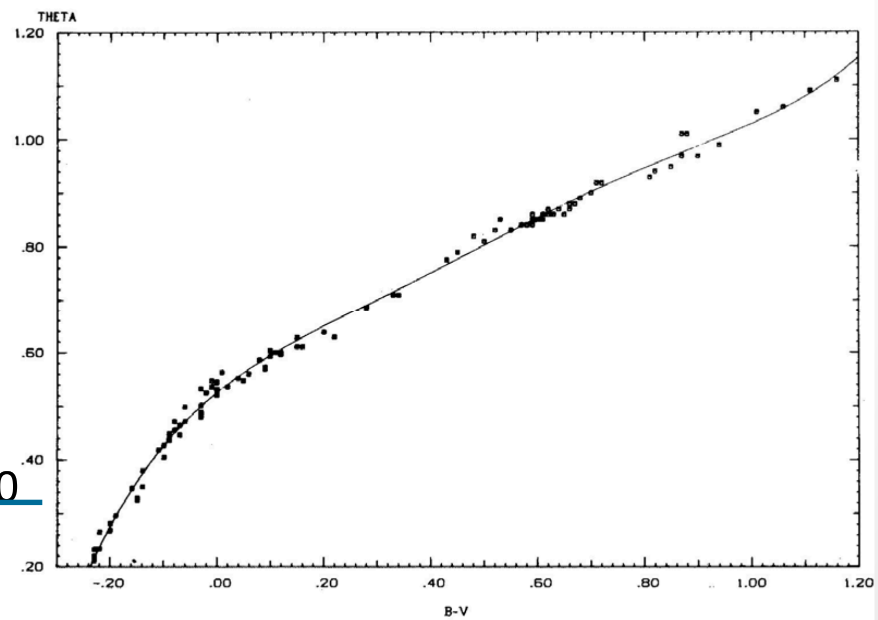
ratios of suitable strong lines
→ T_{eff} or spectral type + calibration T_{eff} *vs* spectral type

excitation equilibrium

line profiles

Source: Bessell 2005





$$\vartheta_{eff} = \frac{5040}{T_{eff}}$$

IRFM: a semi-fundamental T_{eff} scale

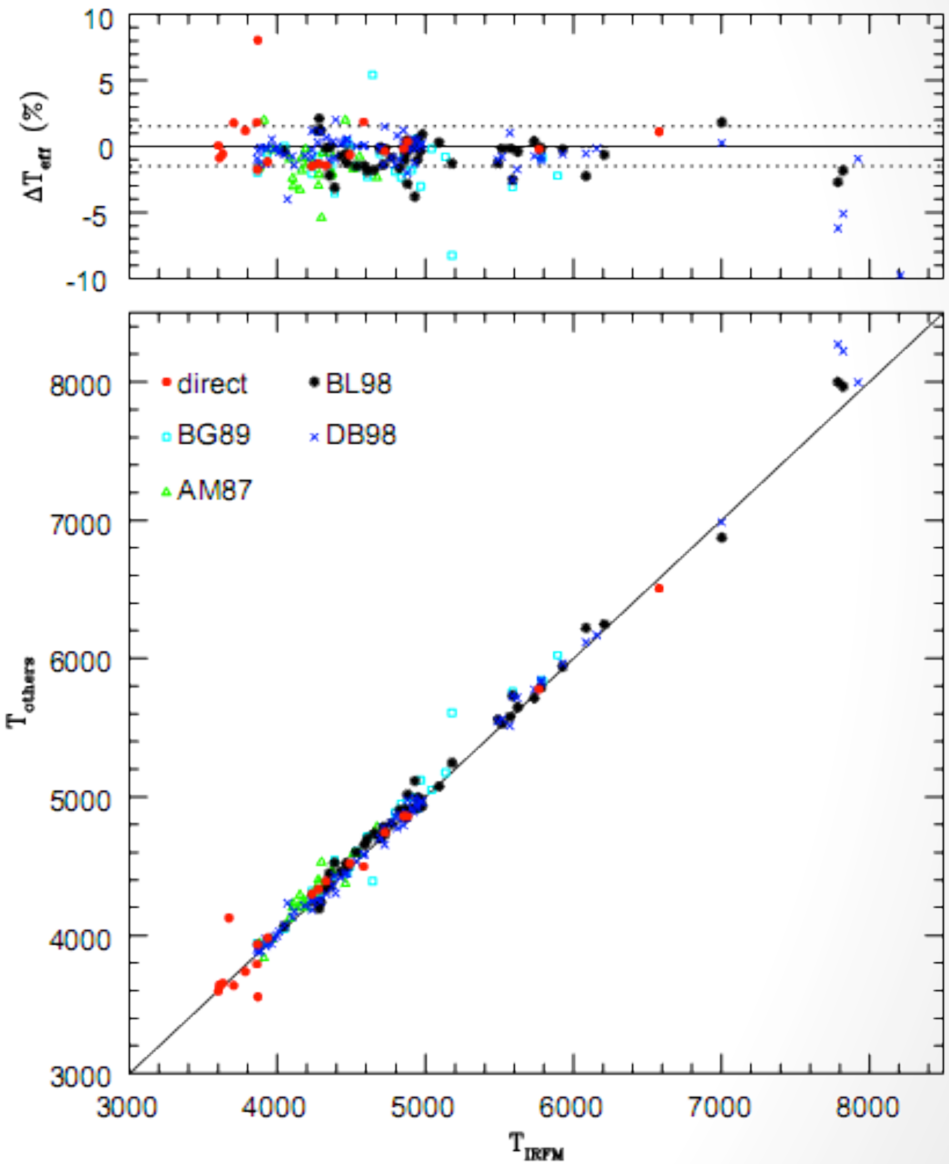
Empirical method, calibrated on the InfraRed Flux Method, gives a relation between photometric colours (like V – I, V – K) and T_{eff}.

Basic idea:

$$\frac{\mathcal{F}\,(\text{surface})}{\mathcal{F}_{\lambda IR}\,(\text{Earth})} = \frac{\sigma\, T_{eff}^{\ 4}}{\mathcal{F}_{\lambda IR}(\text{model})}$$

Once calibrated on stars with known diameters, any colour index can be calibrated on the IRFM.

Comparing different IRFM calibrations, the zero point proves to be uncertain by ±100K, in particular for metal-poor stars.

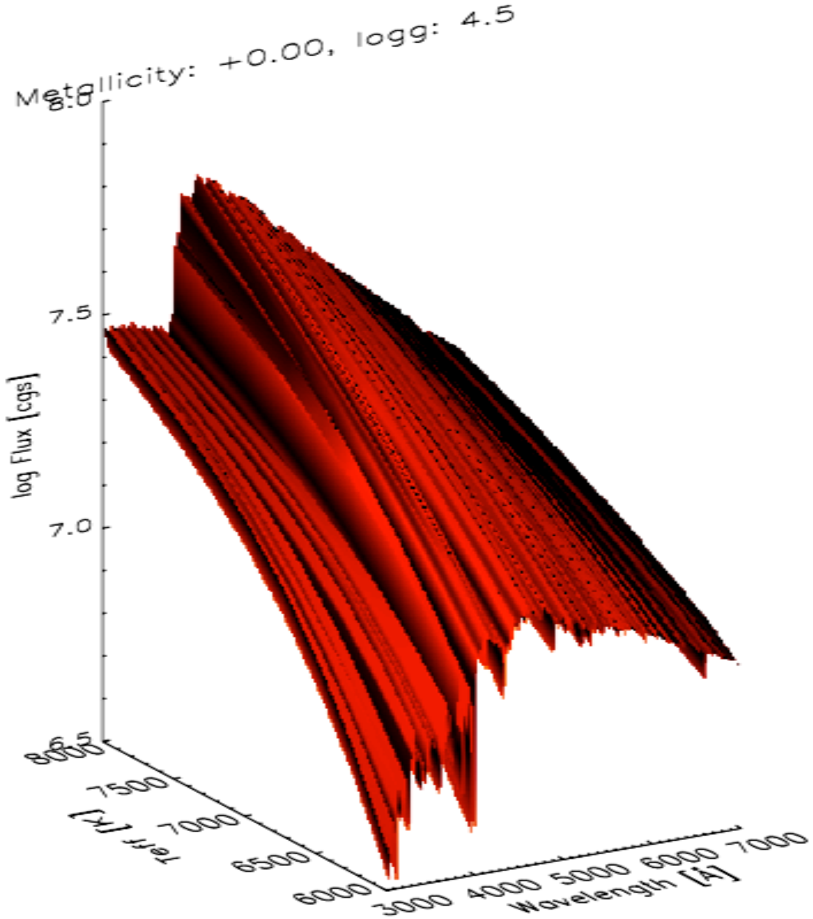


Source: Alonso et al. 1999

Photometry: T_{eff} dependence

T_{eff} variations dominate the flux variations of cool stars

BB approximation: need to measure flux at 2 points to uniquely determine *T*.
In reality, [*m*/Fe] and reddening complicate the derivation of photometric stell. params.

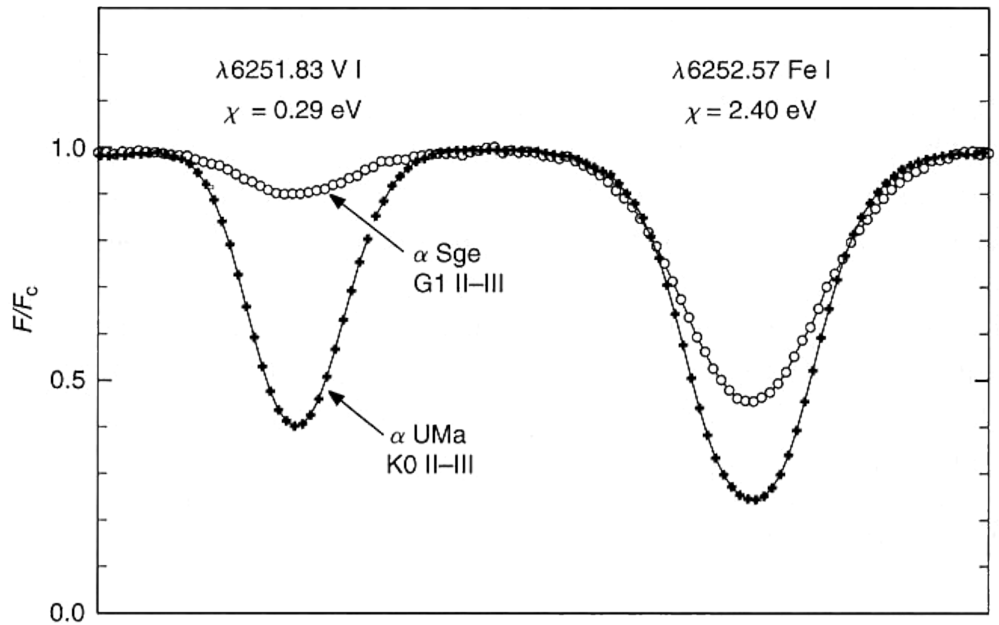


Spectroscopic T_{eff} indicators: line-depth ratios (LDRs)

Ratio of two lines' central depths can be very sensitive to temperature, if the lines are chosen to have different sensitivities to T

Ideally, the LDR is close to 1 and the lines should not be too far apart.

The main challenge lies in a proper T_{eff} calibration across a usefully large part of the HR diagramme



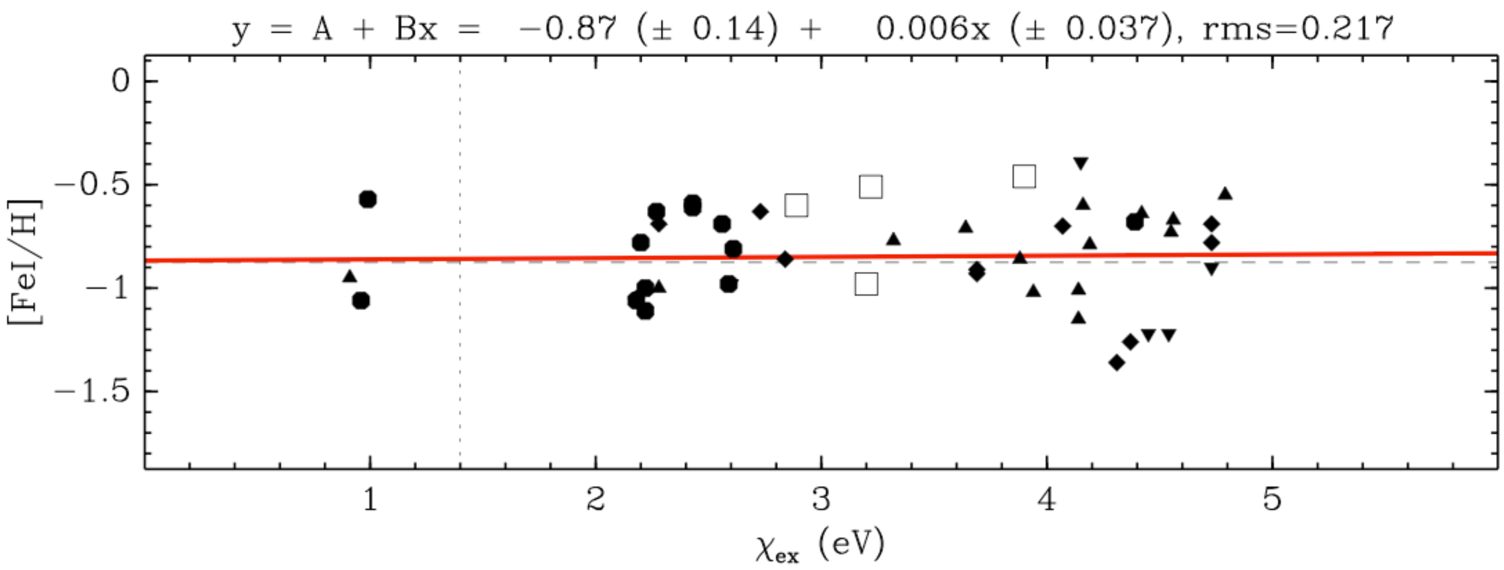
Source: Gray, Fig. 14.7

Spectroscopic T_{eff} indicators : excitation equilibrium

T_{eff} is determined such that the abundance of an element (usually Fe) is independent of the excitation potential (χ_{exc}) of the individual lines*

One needs many lines of a single element sampling a range of χ_{exc} → iron

Final precision depends on spectral resolution, choice and number of lines and S/N ratios



* Boltzmann's eq. under LTE conditions

Source: Letarte, PhD

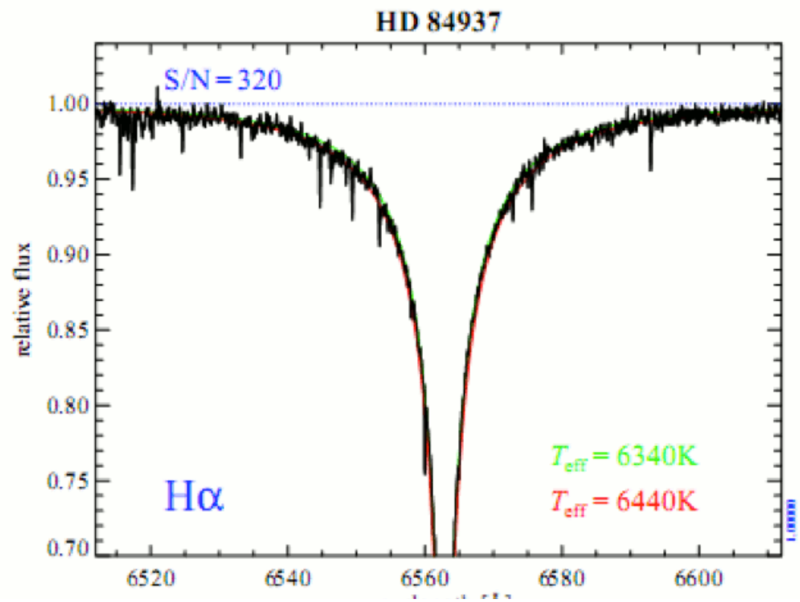
Spectroscopic T_{eff} indicators: H lines

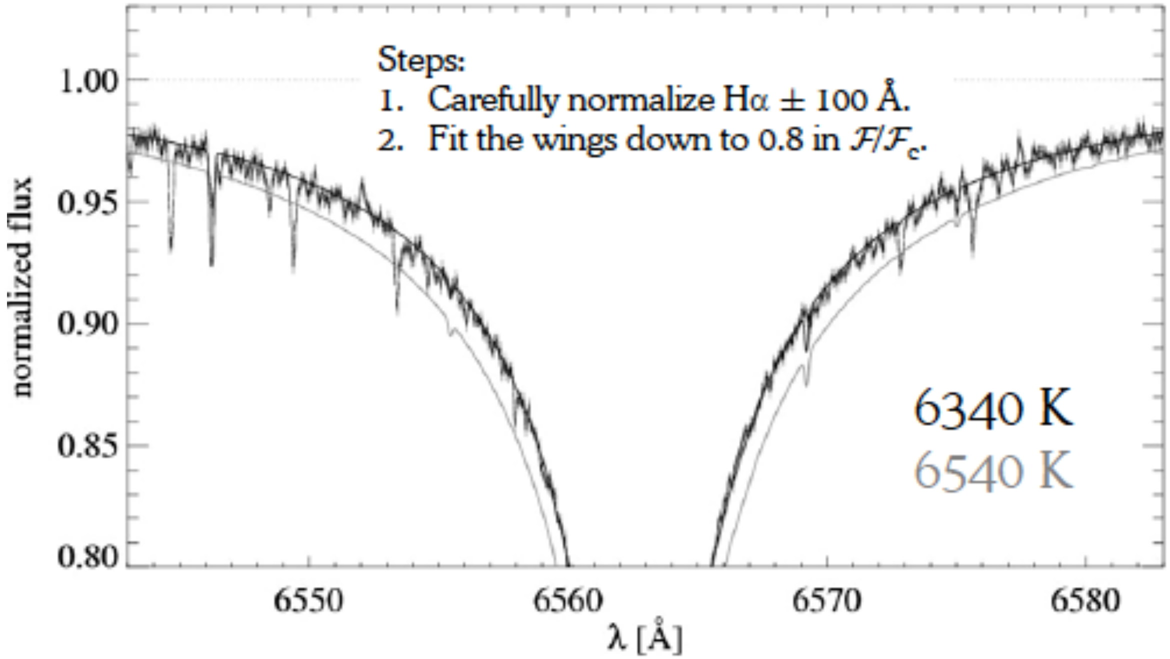
Wings of Balmer lines (above 5000K)

Cool stars: OK
Log *g* and metallicity sensitivity is low, some dependence on the mixing-length parameter (Hβ and higher).

Main challenge: recovering the intrinsic line profiles from (echelle) observations and proper normalization

Hot stars: Balmer lines can constrain the surface gravity





Source: Korn

How to determine log *g*

Calibrated photometry

$$\log g_\star = \log g_\odot + \log \frac{\mathcal{M}_\star}{\mathcal{M}_\odot} + 4 \times \log \frac{T_{\mathrm{eff}\star}}{T_{\mathrm{eff}\odot}} + 0.4 \times (M_{\mathrm{Bol}\star} - M_{\mathrm{Bol}\odot})$$

Parallaxes L, R, (M) → g

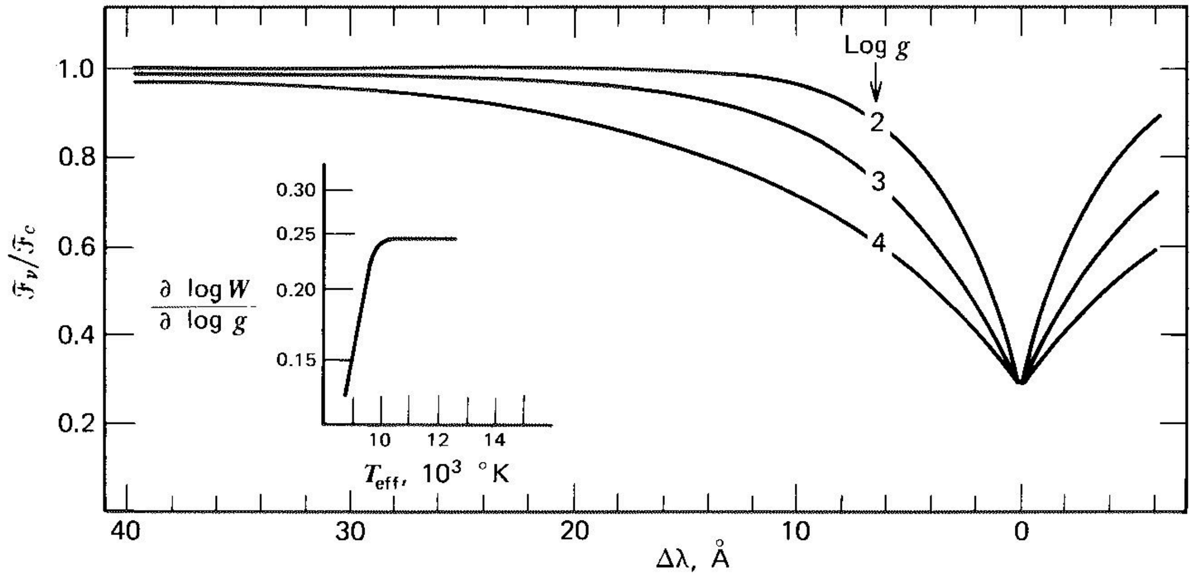
Spectroscopy

Ratio of, e.g., Fe II to Fe I lines

Line profiles:

B-to-F-type stars: Balmer

Late-type stars: strong metallic lines with wings (e.g. Na ID doublet, Ca II IR triplet)



But there is always a temperature dependence – this must be secured beforehand or a pressure-temperature solution must be made.

Photometric log *g*

Largest gravity sensitivity: Balmer jump at 3647Å

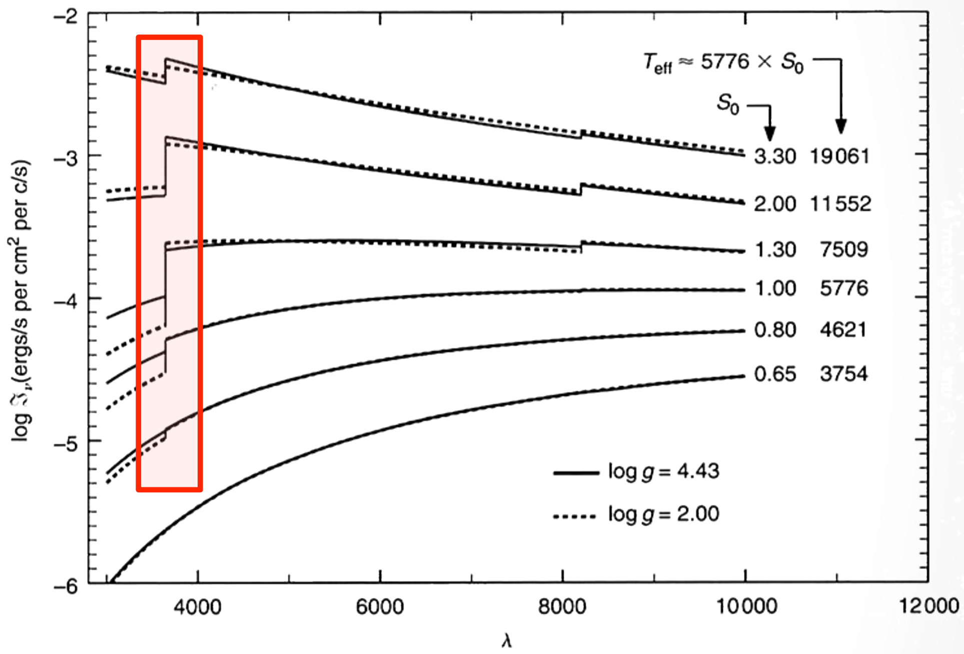
Colour indices like (*U* –*B*) or (*u* –*y*) measure the Balmer discontinuity

Disadvantages:

high line density in spectral region (missing opacity problem)

difficulties with ground-based observations in the near-UV

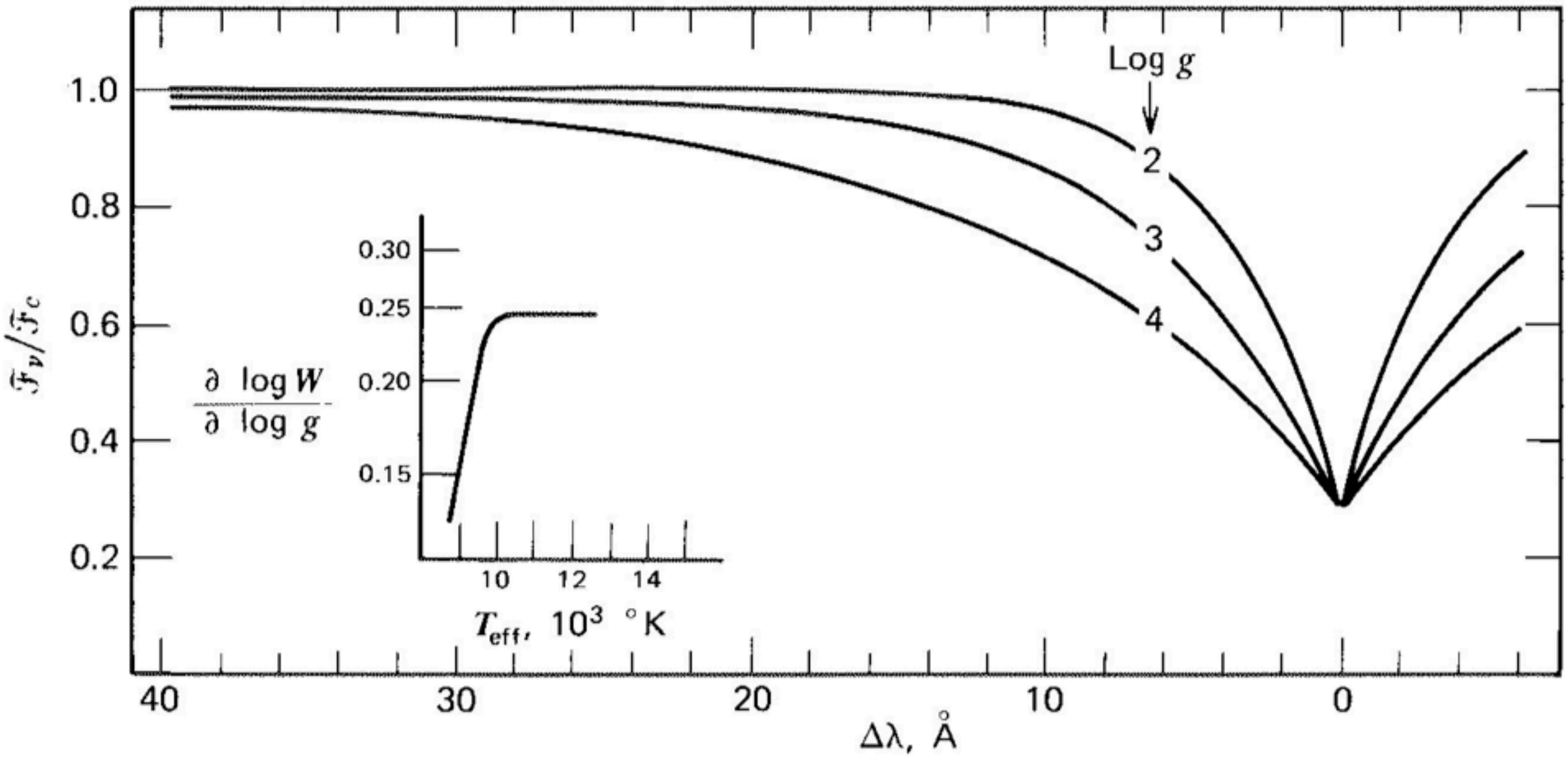
The c1 index [(*u* –*b)*–(*b* –*y*)] works well for metal-poor giants (Önehag *et al*. 2008)



Source: Gray, Fig.10.8

Spectroscopic log *g*: Balmer Lines

Hγ: pressure indicator for T_{eff} > 7500K



Spectroscopic log *g*: ionization balance

Measure of elements in two ionization states (e.g., FeI & Fe II): both should yield same abundance, A.

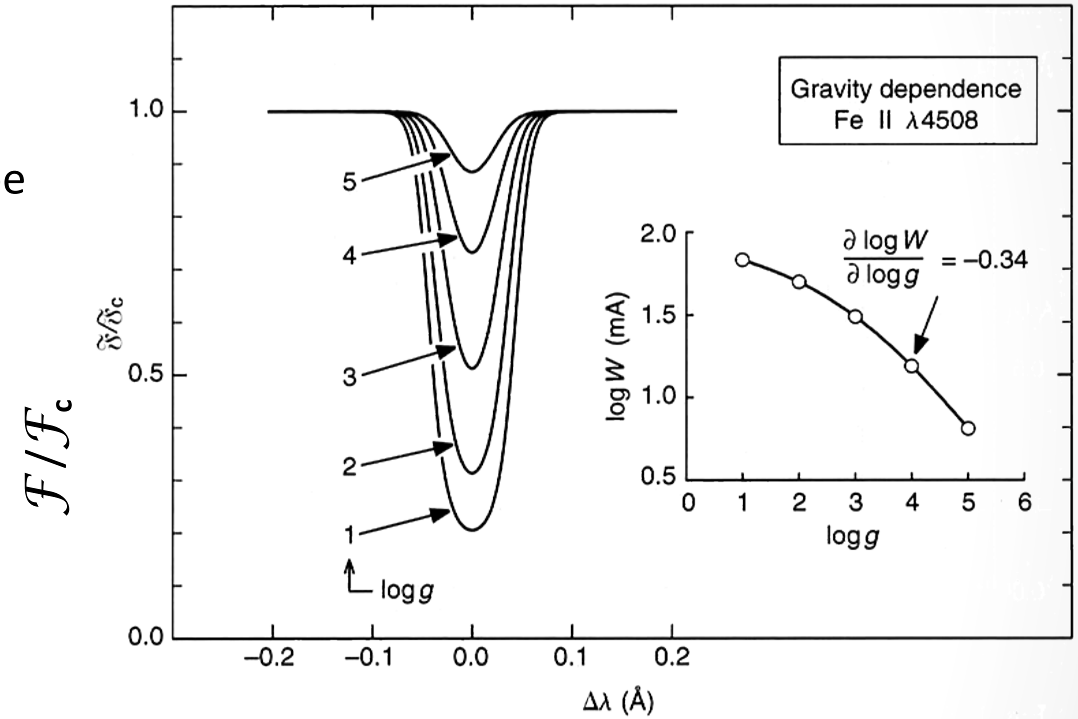
Gravity is a free parameter, to be varied until equality is reached.

By definition, gravity is related to $P_g \propto g^{2/3}$ and $P_e \propto g^{1/3}$

In cool stars:

- Fe I, the dominant species, will depend on $1/P_e$
- Fe II, the minority species, with majority of atoms in state i – 1 = 1, will depend on $1/P_e$

Hydrostatic equilibrium $dP/d\tau_\nu = g/\kappa_\nu$



$\Delta\log\varepsilon$ = 0.1dex → $\Delta\log g$ = 0.3dex (0.1dex ≈ line-to-line scatter)

Spectroscopic log *g*: the strong line method

Damped (neutral) lines show a strong gravity sensitivity, because

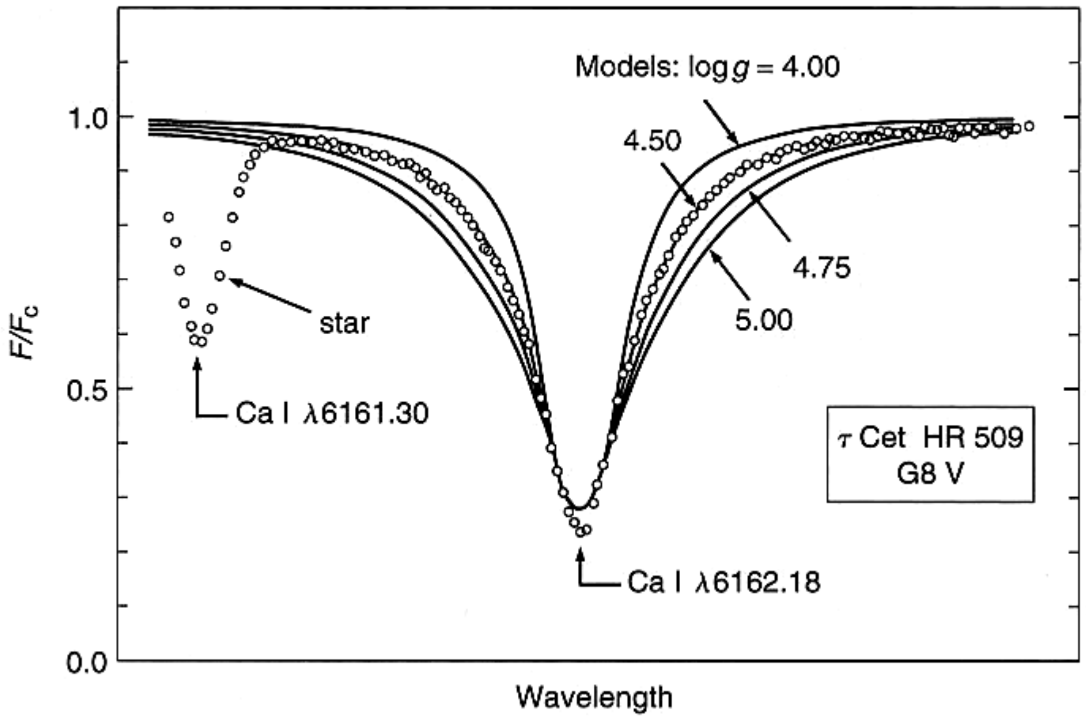
$L_\nu \propto \gamma_6 \propto P_g / g^{2/3}$

Like with ionization equilibria, log ε needs to be known

(best if obtained from weak lines of the same ionization stage, preferably originating from the same lower state:

CaI 6162, FeI 4383, MgI 5183, CaI 4226)

Source: Gray, Fig. 15.4



Below [Fe/H] ≈ -2, there are no optical lines strong enough to serve as a surface-gravity indicator

How to determine the microturbulence velocity, ξ

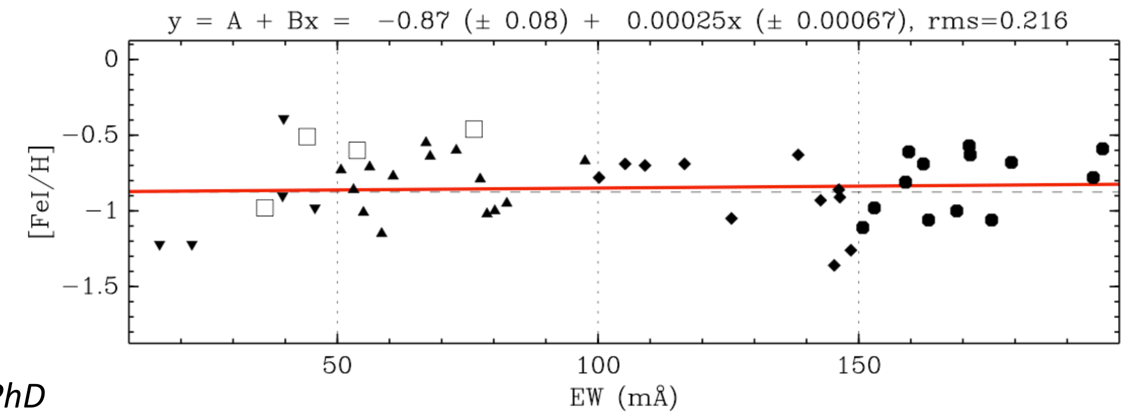
Observed EW of saturated lines > predicted values using thermal and natural broadening alone → extra broadening introduced, the micro-turbulent velocity ξ (fudge factor)

Caused by small cells of motion in the photosphere (treated like an additional thermal velocity in the line absorption coefficient)

No effect on weak lines: these are gaussian, broadening them also makes them shallower → EW is preserved

Can be important in strong lines, by broadening and hence de-saturating them.

It is usually determined by ensuring that for individual elements EW is independent of line. Typical values are 1-2 km/s.



Source: Letarte, PhD

How to determine the metallicity

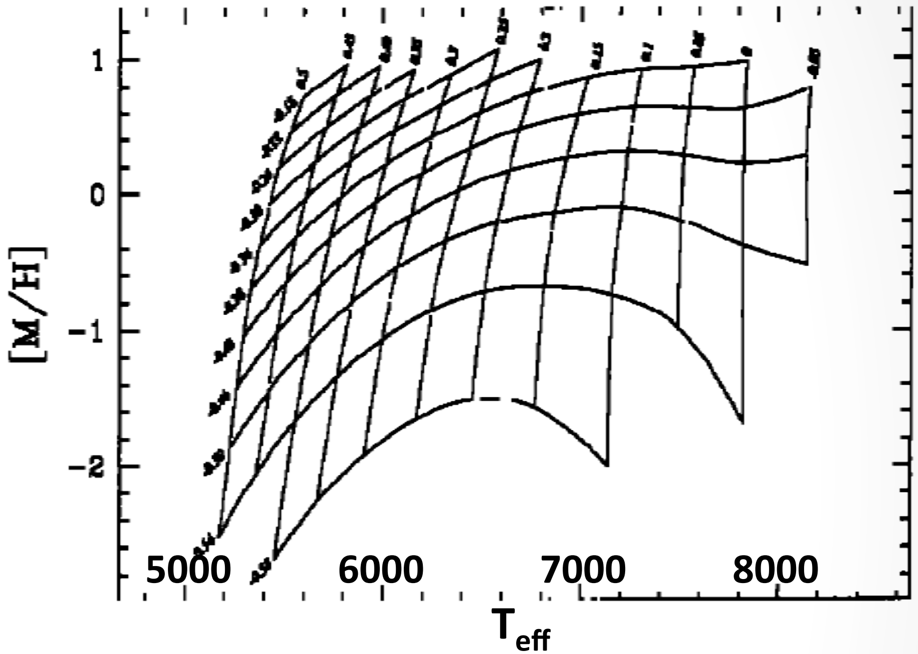
Calibrated photometry

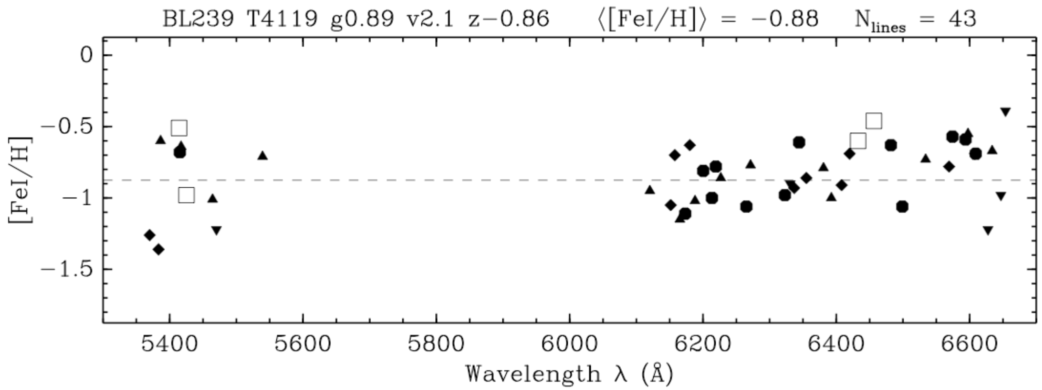
After T_{eff} (and maybe reddening) the *global* metallicity has largest influence on stellar flux

Difficult for stars with [Fe/H] < –2 (e.g. $\delta(U-B)$ loose sensitivity)

Limited precision: ≈0.3 dex

Lines iso-(B2-V1) and iso-m2 of the Geneva photometric system





Spectroscopy

Equivalent widths

Pros *vs.* Cons

Photometry

✓an efficient way of determining stellar parameters
✓can probe very deep
✓freely available (surveys!)
✓comparatively cheap to obtain.

✗ limited (which parameters can be derived)
✗ subject to extra parameters (reddening!)
✗ subject to parameters that cannot be determined well (ξ, [α/Fe]).

Spectroscopy

✓a way of determining a great number of stellar parameters
✓the key technique for obtaining detailed chemical abundances
✓(usually) reddening-free.

✗ comparatively costly at the telescope
✗ currently limited in *V*
✗ more complex to master

- Stellar atmospheres
- Characteristics of star → stellar parameters
- **From lines to abundances**
- Lines (atomic and/or molecular)
- Model atmospheres
- Available tools

From spectral lines to abundances

Curve of growth

Spectrum synthesis

Differential analysis:
→Comparison of one star to another
→Ratio of abundances
→Reference star is necessary

From spectral lines to abundances

Different methods and tools, but …

Need to relate intensity/strength of an observed line to amount of corresponding element in original stellar gas, i.e. need to find the <u>number of absorbing atoms per unit area</u> (N_a) that have electrons in the proper orbital to absorb a photon at the wavelength of the spectral line

Boltzmann and Saha equations are applied and combined with the pressure and temperature of the gas to derive an abundance of the element (i.e. to calculate excitation and ionization):

Saha's equation

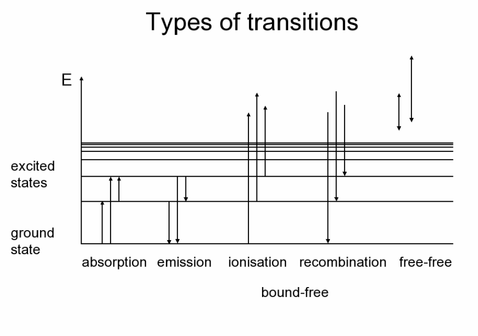
$$\frac{n_1}{n_0} P_e = \frac{(2\pi m_e)^{3/2}(kT)^{5/2}}{h^3} \frac{2u_1(T)}{u_0(T)} e^{-I/kT}$$

It gives the relative number of atoms in two ionization states as a function of electron density n_e and temperature T

Boltzmann's equation

$$\frac{N_b}{N_a} = \left(\frac{g_b}{g_a}\right)\left(e^{-(E_b-E_a)/kT}\right)$$

It gives the population of energy levels *a* and *b*



From spectral lines to abundances

However, not all transitions between atomic states are equally likely. Each transition has a relative probability, or *f*-value (also called *oscillator strength*).

Can be calculated theoretically or measured in a lab

Number of atoms lying above each cm^2 of photosphere: $N_a \times f$-value

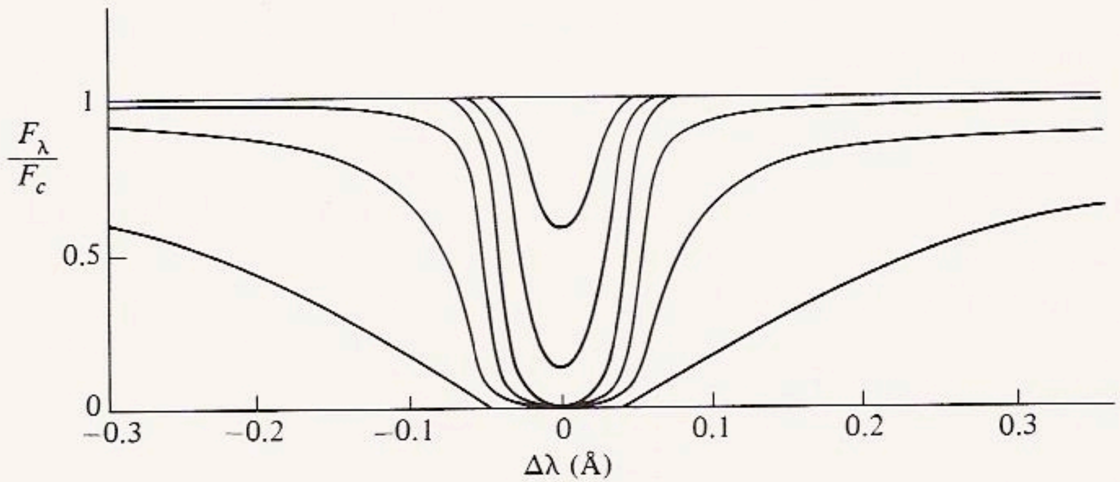
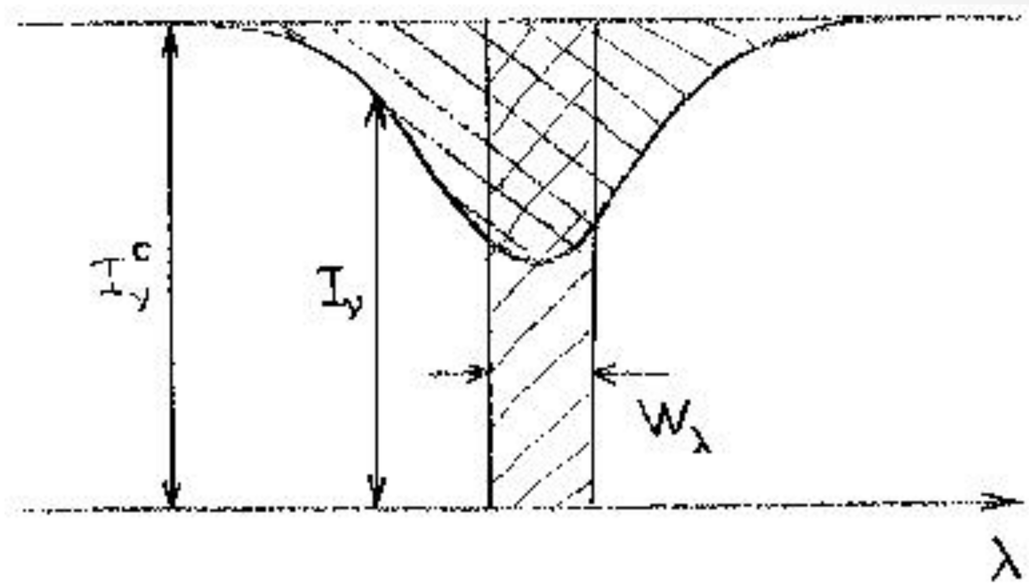


Figure 9.20 Voigt profiles of the K line of Ca II. The shallowest line is produced by $N_a = 3.4 \times 10^{11}$ ions cm^{-2}, and the ions are ten times more abundant for each successively broader line. (Adapted from Novotny, *Introduction to Stellar Atmospheres and Interiors*, Oxford University Press, New York, 1973.)

Measuring abundances: equivalent width

Equivalent width:

$$W_\lambda = \int_0^\infty \frac{I_c - I_\lambda}{I_c}\, d\lambda$$



Geometrically:

W_λ =width of a rectangular, completely opaque line

For a 2-component atmosphere (continuum source + cool layer of depth h), the rectified line profile is:

$$r(\lambda) = \frac{I_\lambda(0)\, e^{-[\kappa_\ell(\lambda)+\kappa_c(\lambda)]h}}{I_\lambda(0)\, e^{-\kappa_c(\lambda)h}} = e^{-\kappa_\ell(\lambda)h}$$

Then $W_\lambda = \int_0^\infty [1 - r(\lambda)]\, d\lambda$

For faint lines: $W_\lambda \approx \int_0^\infty \kappa_\ell(\lambda)\, h \cdot d\lambda = Nh \int_0^\infty \alpha_\lambda\, d\lambda = Nhf\, \frac{\pi e^2}{m_e c}\, \frac{\lambda^2}{c} \propto Nhf\lambda^2$

where N is the number density of atoms able to absorb the incident radiation

Measuring abundances: curve of growth

Tool to determine N_a → abundance (EW varies with N_a)

Weak lines: linear part $W \propto A$

Doppler core dominates and the width is set by the thermal broadening $\Delta\lambda_D$. Depth of the line grows proportionally to abundance A

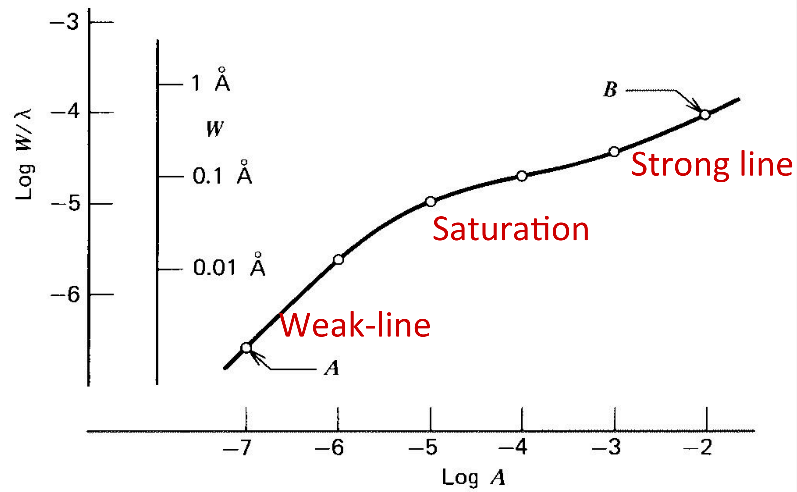
Saturation: plateau $W \propto \sqrt{\log A}$

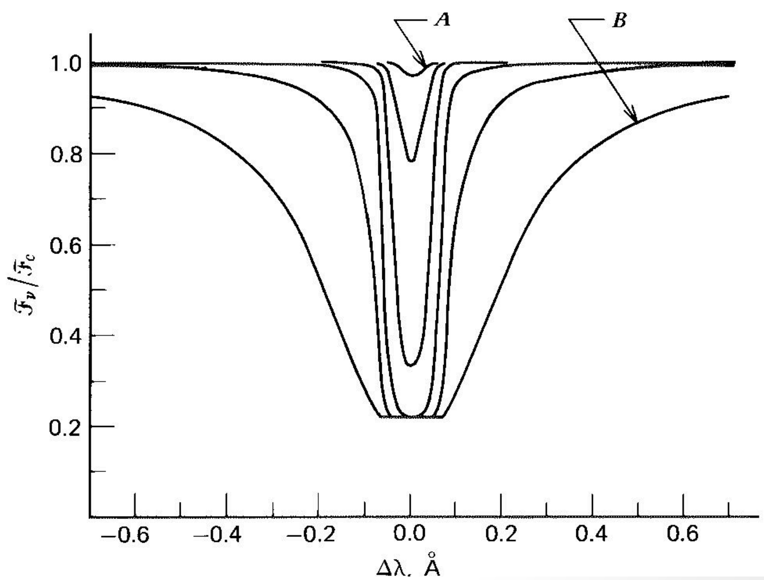
Doppler core approches max. value and line saturates towards a constant value

Strong lines: damping wings dominate $W \propto \sqrt{A}$

optical depth in wings becomes significant compared to κ_ν. Strength depends on g, but for constant g the EW is proportional to $A^{1/2}$

Small/weak lines are best for abundance determination





CoG dependencies: temperature

Temperature can affect line strength via:

- N_r/N_E, κ_ν, θ_{ex}

CoG shape looks the same, only shifted for different values of the excitation potential

Fewer atoms are excited to the absorbing level when χ higher.

Amount of each shift can be interpreted as $\theta_{exc}\chi$.

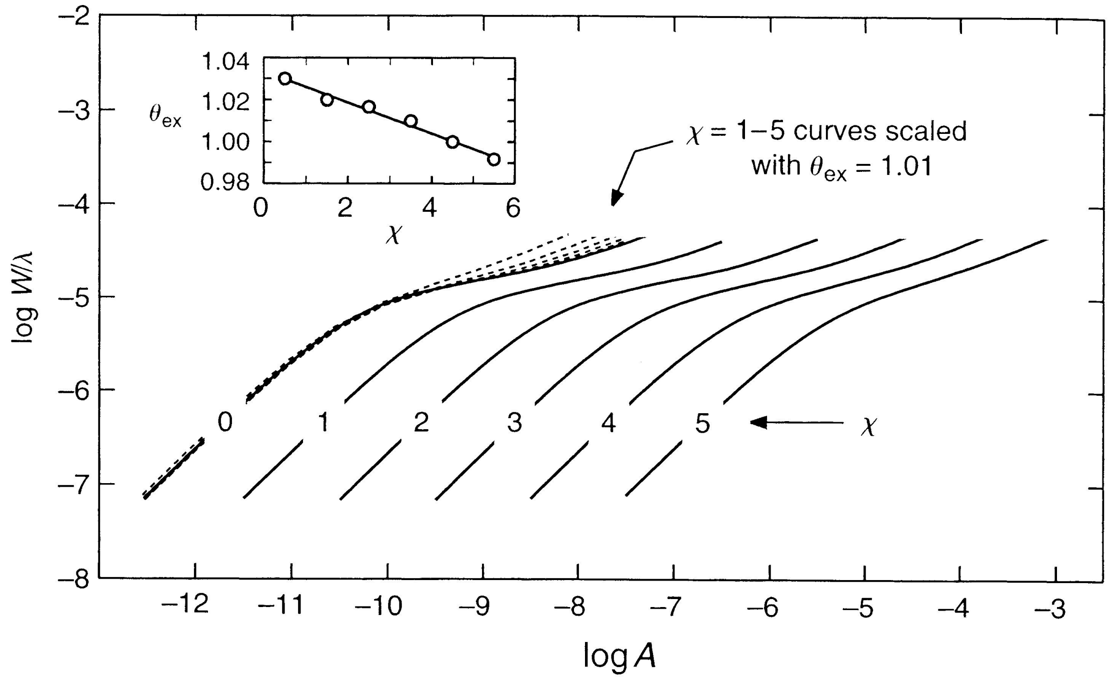


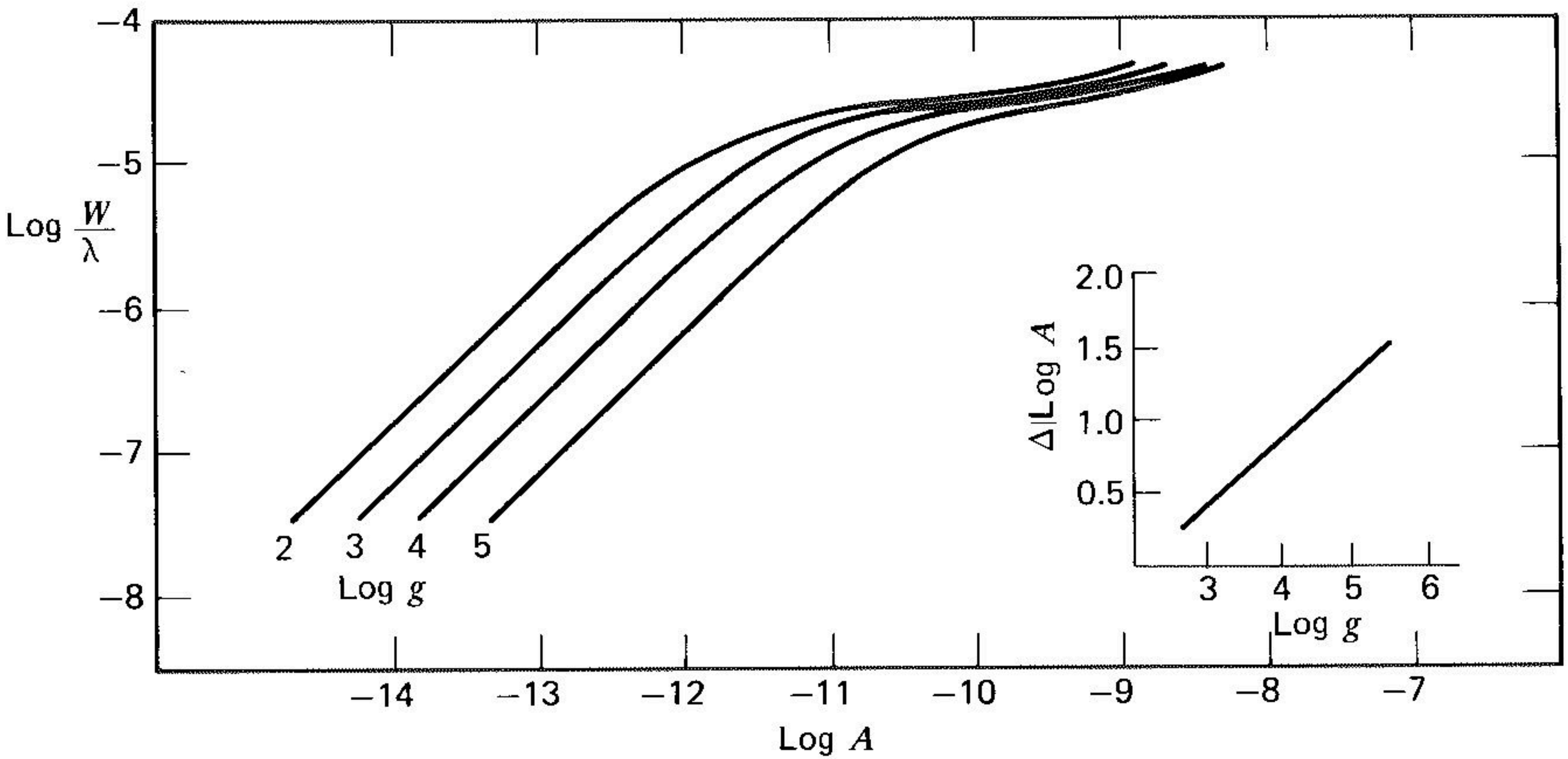
Fig. 16.1. Curves of growth shift to the right as the excitation potential is increased. These curves are computed for Fe I λ6253 in a solar-temperature model with a surface gravity of 10^4cm/s^2. When the shifts are translated into θ_{ex} according to Eq. (16.4), θ_{ex} is found to vary slightly as shown in the inset graph; higher excitation lines are formed deeper where the temperatures are higher. The dashed curves show scaling for a fixed θ_{ex} of 1.01, and we see that the differences in the linear portion are considerably smaller than observational errors. Differences in development of the strong-line portion of the curves stems from the increase in van der Waals damping with increasing excitation potential.

CoG dependencies: gravity

Gravity can affect line strength through:

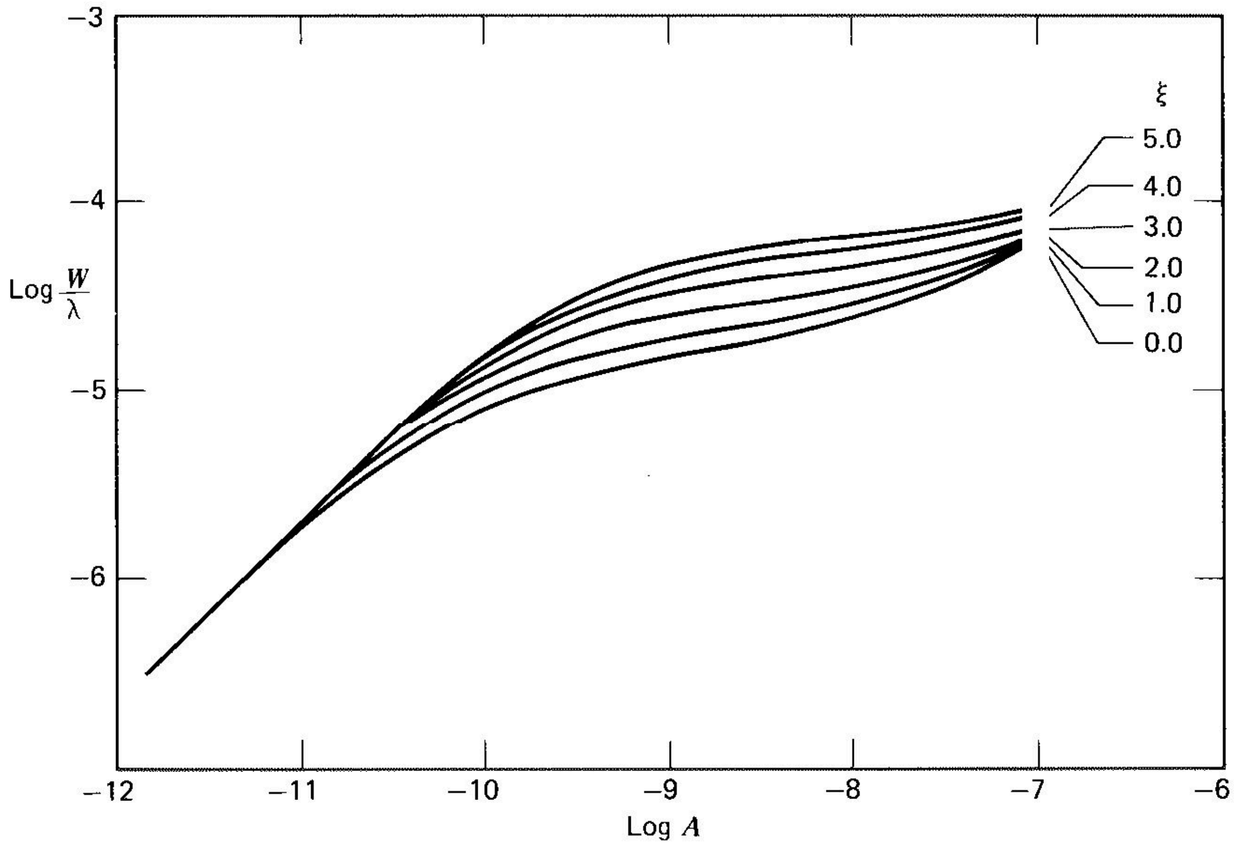
- N_r/N_E (if increases, A decreases at given W/λ)
- κ_ν

Since both of these can be sensitive to the pressure, for neutral lines the effects cancel → effect important only for ionized species



CoG dependencies: microturbulence

The presence of microturbulence delays saturation by spreading the line absorption over a larger spectral band.



CoG analysis for abundances

Advantages:

Simple, you measure the equivalent width of a line and read the abundance off the log W *vs* log A plot

Disadvantages:

Lots of calculations
Difficulty in dealing with microturbulence and saturation effects:

> Make an initial guess of ξ
> Theoretical cogs are calculated for all measured EWs of some element with lots of lines
> From each line one derives an abundance A and plots it *vs* W
> If A is found to be a function of W → ξ must be wrong
>
> One happily choose a new ξ and start all over
>
> This must continue until one finds convergence

CoG: principles of abundance estimate

1. Compute a theoretical cog for a given line of a given atom/ion

Observe W of this line → log A_0

This is what codes do implicitly for each line

2. If one doesn't want to compute more than 1 cog:

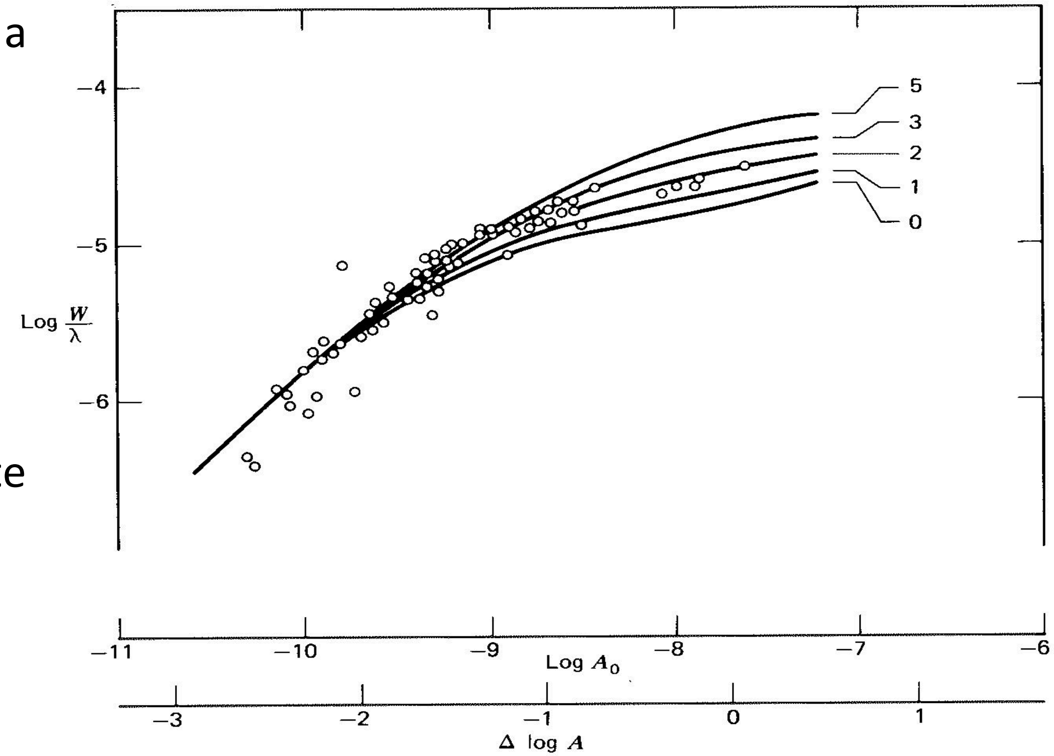
Observed W of other lines

Then plot log(W/λ) vs Δlog A

Then empirical cog

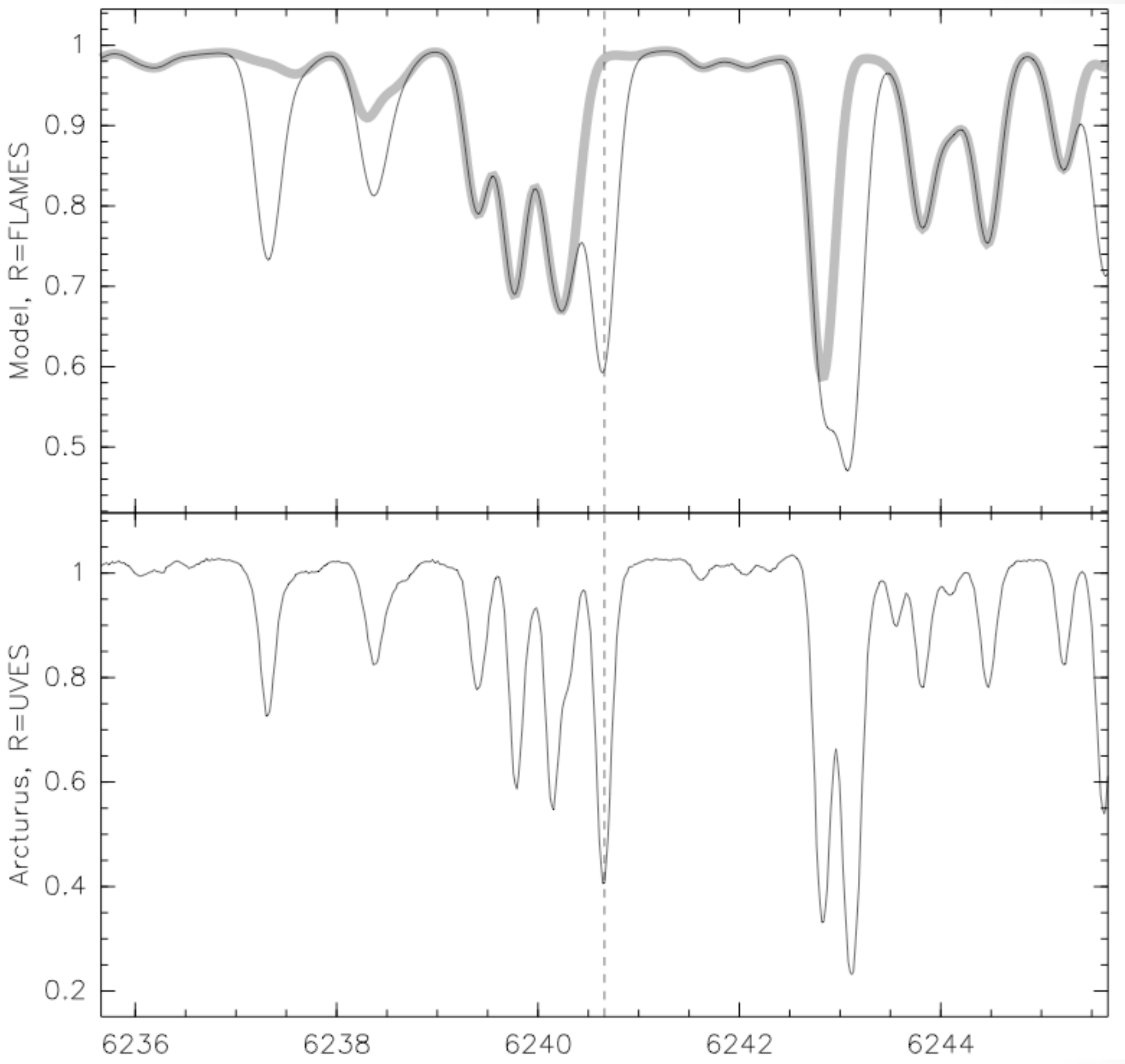
Then shift curve onto theoretical one

→ shift=log A_0



$$\Delta \log A = \log\left(\frac{gf\lambda}{g_0 f_0 \lambda_0}\right) - \theta_{exc}(\chi - \chi_0)$$

Crowding & Blends



→ Spectrum Synthesis

Spectrum Synthesis

In real life, one no longer does a curve-of-growth analysis, but rather a full spectral synthesis.

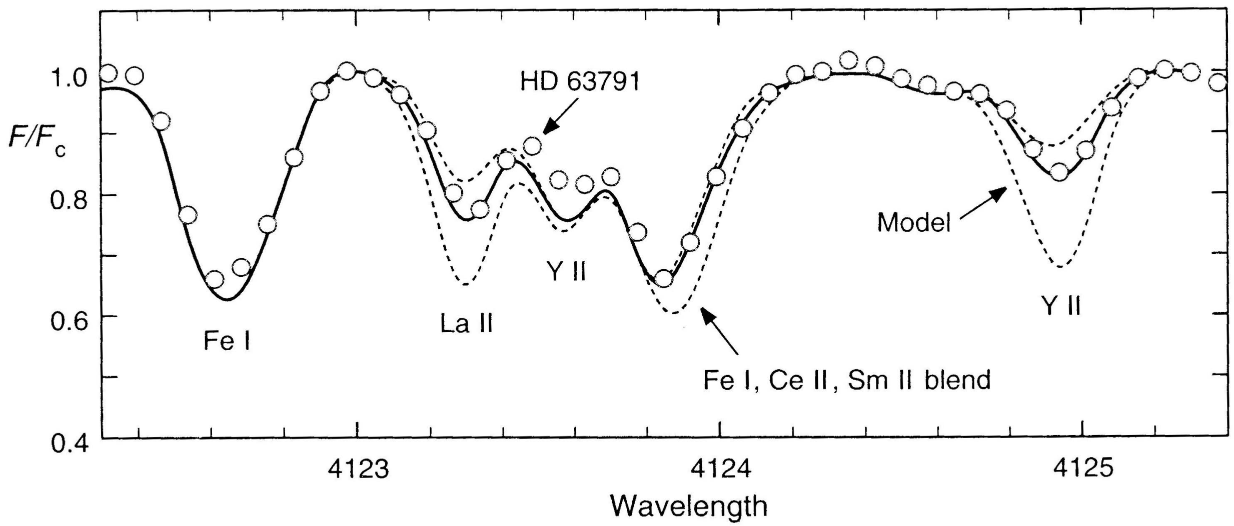
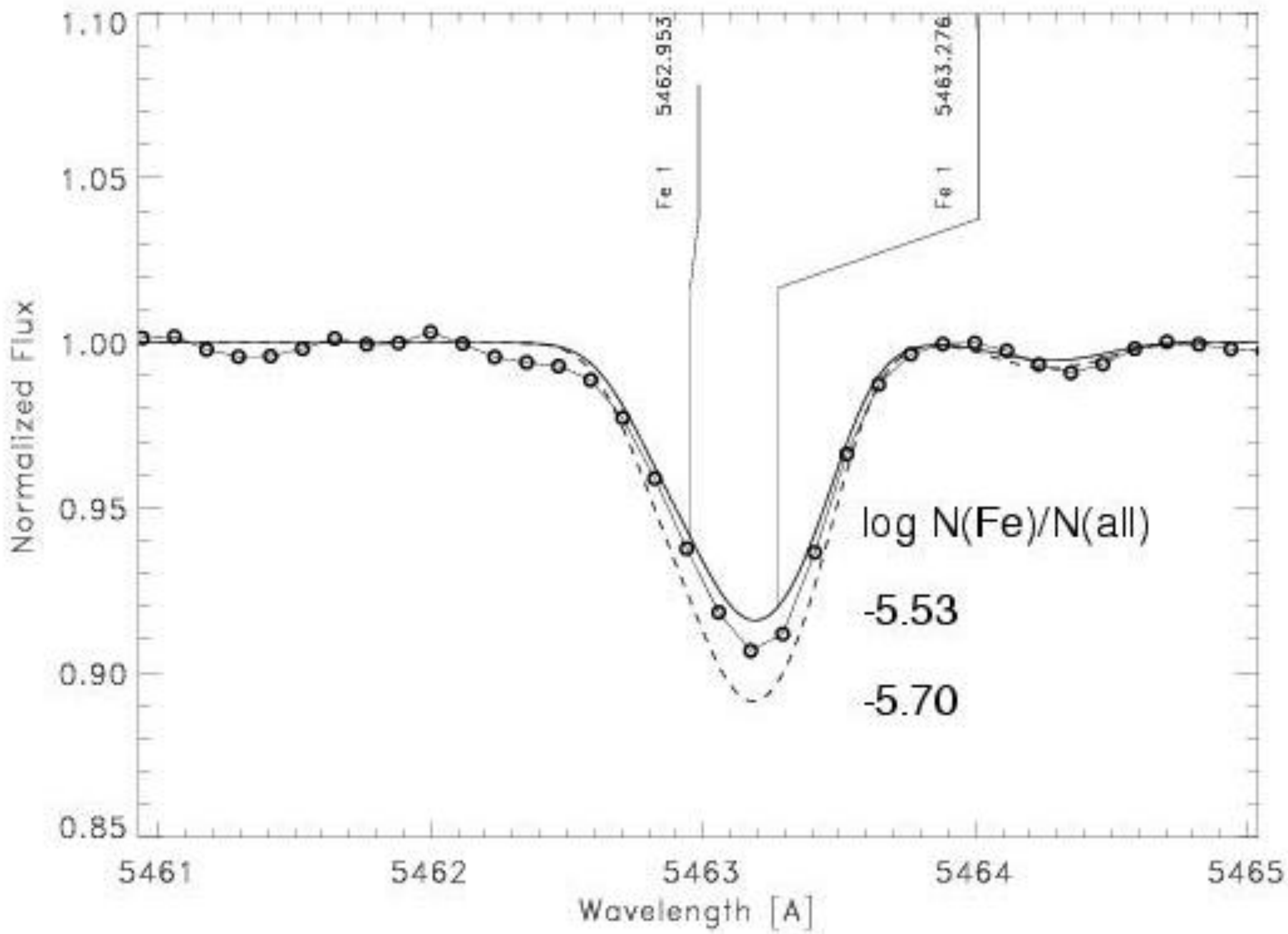


Fig. 16.9. The circles show the observed spectrum, while the lines are for models ($T_{eff} = 4725$ K, log $g = 1.70$, and $\xi = 1.60$ km/s) with different chemical abundances. The solid line is deemed to fit best. Based on data in Fig. 2 of Burris *et al.* (2000). The resolving power is ~20 000 and the signal-to-noise ratio ~100.

Spectrum synthesis



Source: Heiter

Measuring abundances: expected precision

It is difficult to determine the temperature of a star to better than 50–100 K

It is difficult to determine the gravity of a star to better than 0.1-0.2 dex

It is difficult to determine the microturbulent velocity of a star to better than 0.2km/s

Absolute abundances > 0.1-0.2 dex!

Problems

→ Uncertain or wrong log(gf) values

→ NLTE effects

→ 3D hydrodynamic models → Z_{Sun} =0.012 instead of 0.018!

But

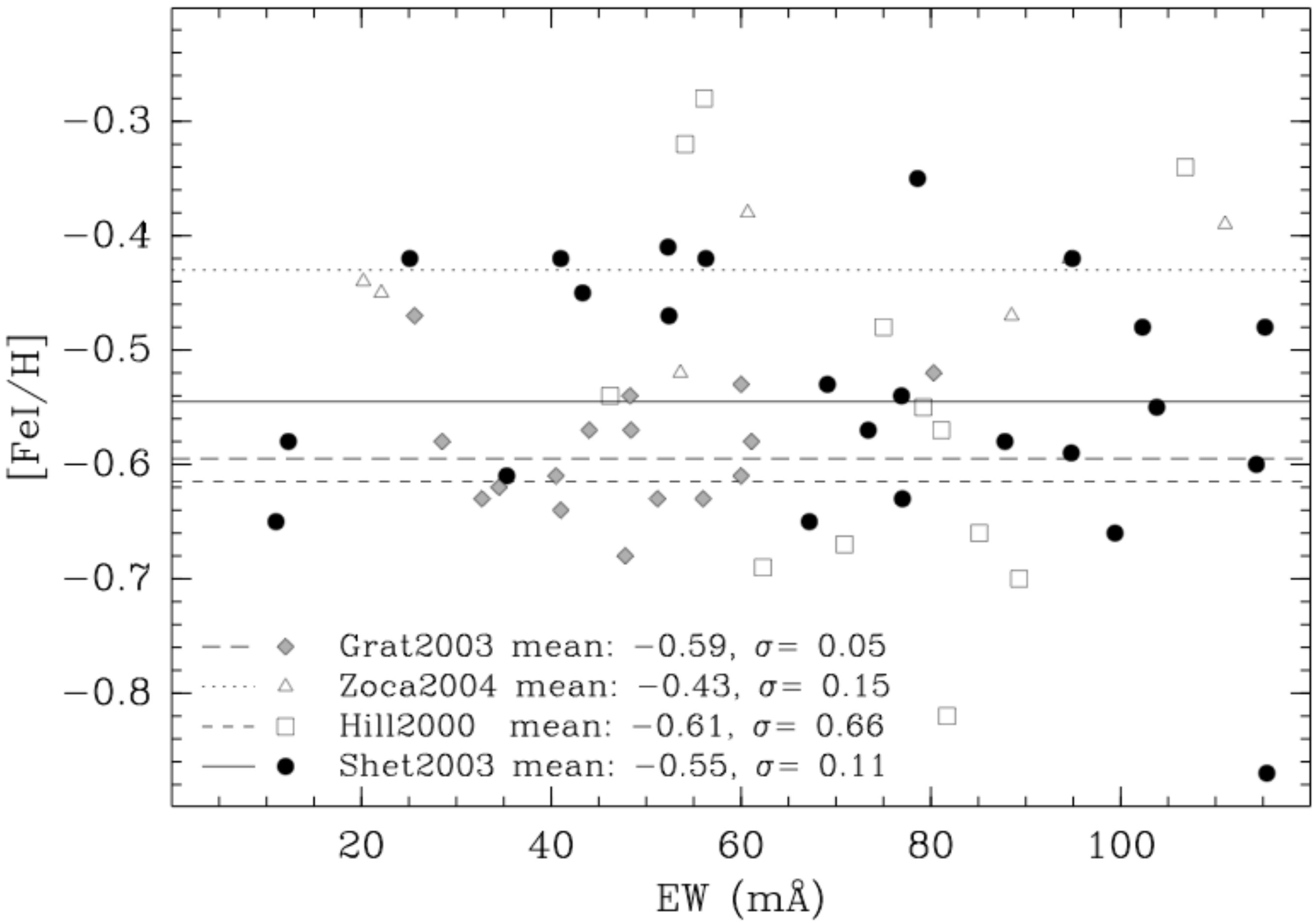
One can work differentially, by taking the abundance ratios between two similar stars (≈ T_{eff}) → uncertainty on the oscillator strengths cancel.

Differential abundances (rel. to the Sun) ≈ 0.04-0.05 dex!!

- Stellar atmospheres
- Characteristics of star → stellar parameters
- From lines to abundances
- **Lines (atomic and/or molecular)**
- Model atmospheres
- Available tools

Line List

A proper line list is a critical part of the analysis, and building one needs some care. Lines need to be chosen carefully, making sure they have reliable gf-values and are sufficiently isolated from their neighbours at the resolution of the observations and of course to lie within the wavelength coverage of the instrument.



Line absorption data

Need to know: λ, $E_{low,}$ $J_{low,}$ f, γ_1, γ_2

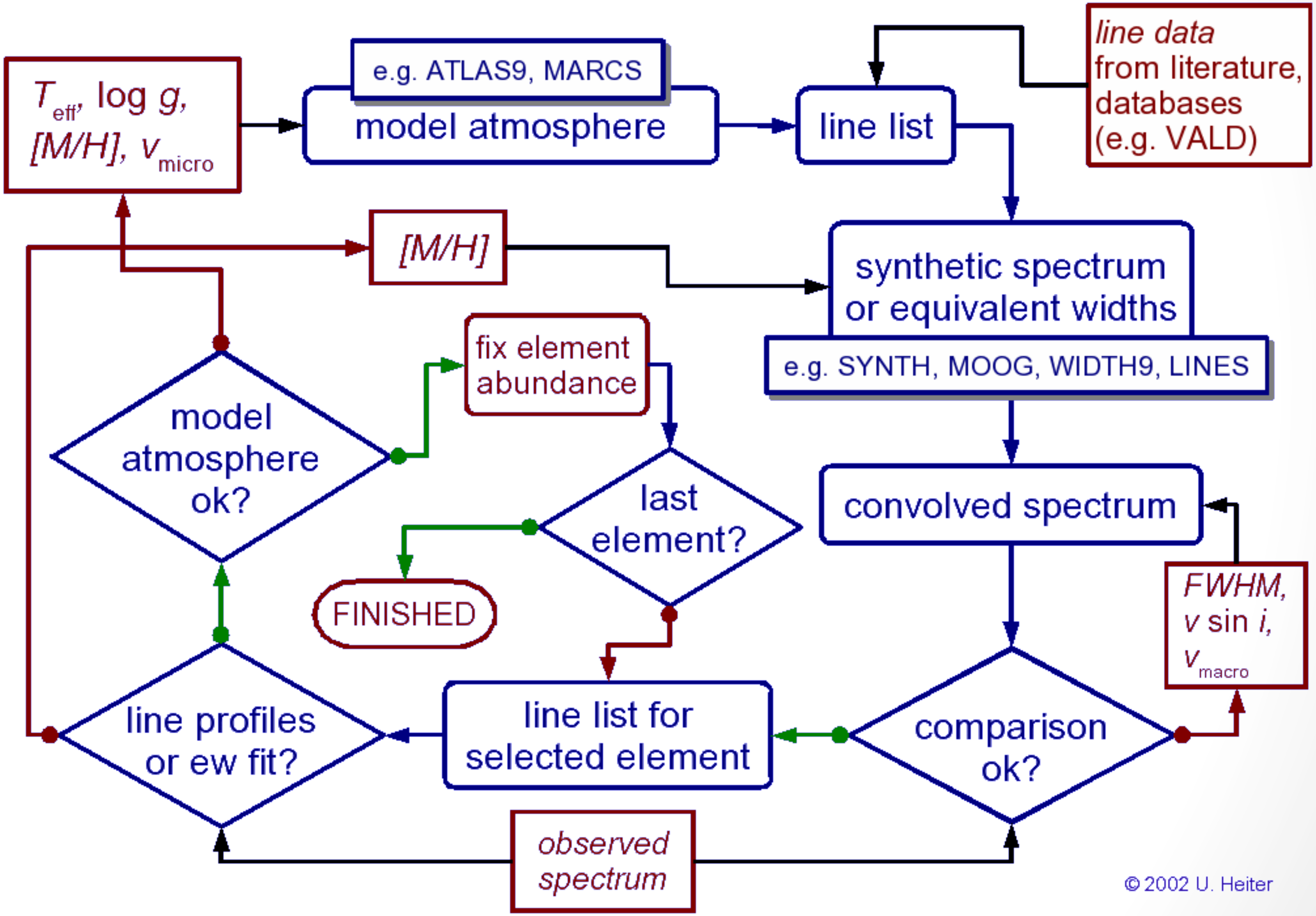
NIST Atomic Spectra Database
http://physics.nist.gov/PhysRefData/ASD/lines_form.html

VALD Vienna Atomic Line Database
http://www.astro.uu.se/~vald/php/vald.php

HITRAN HIgh-resolution TRANsmission molecular absorption database
http://www.cfa.harvard.edu/HITRAN/

Not to forget all the missing (or not well defined) lines ....

Abundance analysis



- Stellar atmospheres
- Characteristics of star → stellar parameters
- From lines to abundances
- Lines (atomic and/or molecular)
- **Model atmospheres**
- Available tools

What is a model stellar photosphere?

To make the modelling of stellar atmospheres manageable, a variety of assumptions are traditionally made about stellar photospheres

Model atmosphere

- Plane parallel layers along depth **t**
- Parameters:
  - Gravity — **g** (constant)
  - Effective temperature — T_{eff} — $\mathbf{F} = \sigma \cdot \mathbf{T}_{\mathbf{eff}}^{4}$
  - Chemical composition — **[M/H]**
- Equations:
  - Transport of radiative energy $$\frac{dI}{ds} = \varepsilon - \kappa \cdot I$$
  - Conservation of energy $$\mathbf{F(t)} = \sigma \cdot \mathbf{T}_{\mathbf{eff}}^{4}$$
  - Hydrostatic equilibrium $$\frac{dP}{dt} = g \cdot \rho$$

Source: Heiter

Assumptions and consequences

(1) plane-parallel geometry

all physical variables a function of only one space coordinate

but stars are essentially spherical!

in many cases photosphere is very thin: $\Delta R / R \ll 1$ ($\approx 5 \times 10^{-4}$ Sun)

(2) homogeneity

no fine structures and granularity

but stars with convection, spots, etc … difficult to resolve

average homogeneous model → *average* stellar properties

(3) Stationarity

time-independence of stellar spectra on human timescales

plenty of exceptions here (pulsations, mass loss, SN)

hope to observe *average* stellar properties

Source: RadiativeTransfer in Stellar Atmospheres, R.J. Rutten2003

What is a model stellar photosphere?

(4) hydrostatic equilibrium

no large scale accelerations in photosphere

no dynamical significant mass loss

gravitational force: $dF_{grav} = -G\, M_r dm / r^2 = -g(r)\, dm$

pressure force: $dF_p = -A(P(r+dr) - P(r))$

radiation force: $dF_{rad} = g_{rad} dm$ (mostly important in hot stars $\alpha\ T^4$)

$\Sigma dF i = 0$ → $dP/dr = -\rho(r)\,(\,g(r) - g_{rad}\,)$

(5) flux constancy (radiative equilibrium)

stellar atmospheres are much too cool and tenuous to fuse nuclei

energy coming from core just transported by radiation or convection

total energy output (luminosity): $L = 4\pi r^2 F(r)$ = const.

(6) local thermodynamic equilibrium

existence of a radial temperature gradient → 2 volumes not in eq.

validity of TE <u>locally</u>

Source: RadiativeTransfer in Stellar Atmospheres, R.J. Rutten2003

Model outputs

A 1D model atmosphere is a tabulation of various quantities as a function of (optical) depth

Table 9.2. *Model photospheres.*

$\log \tau_0$	T (K)	$\log P_g$ (dyne/cm^2)	$\log P_e$ (dyne/cm^2)	$\log \kappa_0/P_e$ (cm^2/g per dyne/cm^2)	x (km)
Solar model, $S_0 = 1.0$, $\log g = 4.438$ cm/s^2					
−4.0	4310	2.87	−1.16	−1.22	−509
−3.8	4325	3.03	−1.02	−1.23	−476
−3.6	4345	3.17	−0.89	−1.24	−448
−3.4	4370	3.29	−0.78	−1.25	−422
−3.2	4405	3.41	−0.66	−1.26	−397
−3.0	4445	3.52	−0.55	−1.28	−373
−2.8	4488	3.64	−0.44	−1.30	−349
−2.6	4524	3.75	−0.33	−1.32	−325
−2.4	4561	3.86	−0.23	−1.33	−301
−2.2	4608	3.97	−0.12	−1.35	−277
−2.0	4660	4.08	−0.01	−1.37	−252
−1.8	4720	4.19	0.10	−1.40	−228
−1.6	4800	4.30	0.22	−1.43	−203
−1.4	4878	4.41	0.34	−1.46	−177
−1.2	4995	4.52	0.47	−1.50	−151
−1.0	5132	4.63	0.61	−1.55	−124
−0.8	5294	4.74	0.76	−1.60	−97
−0.6	5490	4.85	0.93	−1.66	−70
−0.4	5733	4.95	1.15	−1.73	−43
−0.2	6043	5.03	1.43	−1.81	−19
0.0	6429	5.10	1.78	−1.91	0
0.2	6904	5.15	2.18	−2.01	15
0.4	7467	5.18	2.59	−2.11	27
0.6	7962	5.21	2.92	−2.18	37
0.8	8358	5.23	3.16	−2.23	46
1.0	8630	5.26	3.32	−2.25	56
1.2	8811	5.29	3.42	−2.27	68

- Stellar atmospheres
- Characteristics of star → stellar parameters
- From lines to abundances
- Lines (atomic and/or molecular)
- Model atmospheres

- **Available tools**

Data and tools needed

Data needed:

– **Model atmospheres** (esp. T-τ relation) for various T_{eff}, log*g* and chemical compositions

- Kurucz models (ETL, plane parallel): very extended grid,
  http://kurucz.harvard.edu/grids.html

- (OS)MARCS models (ETL, plane parallel/spherical): for cool stars (4000 to 8000 K)
  http://marcs.astro.uu.se/

- TLUSTY models (NLTE, plane parallel): for hot stars (27500 to 55000 K)
  http://nova.astro.umd.edu/Tlusty2002/tlusty-grids.html

– **Line data**: wavelengths, excitation potentials, oscillator strengths, broadening parameters

http://www.astro.uu.se/~vald/php/vald.php

– **Observed spectrum** normalized to the continuum, and equivalent widths

Data and tools needed

Tools needed:

– Data reduction software

- Iraf, MIDAS and/or instrument-specific pipelines, IDL

– Good procedure for continuum normalization → not so easy (e.g. Hα)

– Code of spectral synthesis, e.g.:

ATLAS/SYNTHE (Kurucz)
http://kurucz.harvard.edu

Moog (Sneden) for average and cool stars
http://verdi.as.utexas.edu/moog.html

(OS)MARCS (Uppsala/B. Plez suites)
http://marcs.astro.uu.se/

SME (Valenti & Piskunov)
http://tauceti.sfsu.edu/Tutorials.html

Synspec (Hubeny & Lanz) for hot stars
http://nova.astro.umd.edu/Synspec43/synspec.html

MARCS Stellar Models

<u>Plez 2000-2002</u>

- Geometry: Plane-parallel approximation
- Temperature: $3800 \leq T_{\rm eff} \leq 5200\,{\rm K}$ in steps of 200 K
- Gravity: $0.5 \leq \log g \leq 4.5$ dex in steps of 0.5 dex
- Metallicity: $-4.0 \leq [{\rm Fe/H}] \leq -1.00$ dex in steps of 0.25 dex
- Alpha: Enhanced, $[\alpha/{\rm Fe}] = 0.4$

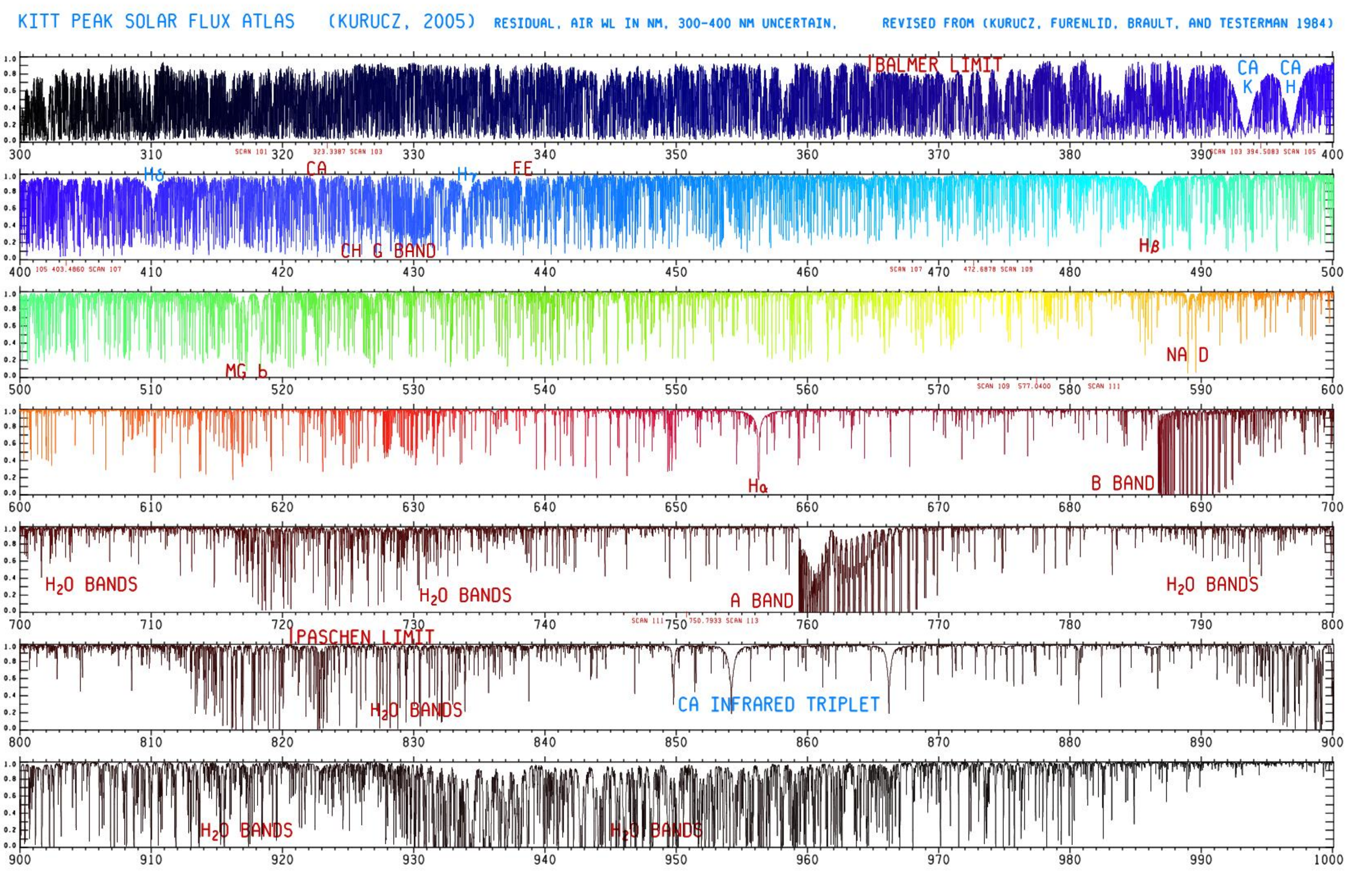
<u>MARCS 2005</u>

- Geometry: Spherical
- Temperature: $4000 \leq T_{\rm eff} \leq 5500\,{\rm K}$ in steps of 250 K
- Gravity: $0.0 \leq \log g \leq 3.5$ dex in steps of 0.5 dex
- Metallicity: $-1.5 \leq [{\rm Fe/H}] \leq +1.00$ dex in steps of 0.25 dex
- Alpha: Standard, $[\alpha/{\rm Fe}] = 0$ at $[{\rm Fe/H}] = 0$, +0.1 for each -0.25 dex until it reaches +0.4 at $[{\rm Fe/H}] \leq$ -1.0.

<u>Plez 2005</u>

- Geometry: Spherical
- Temperature: $3600 \leq T_{\rm eff} \leq 4000\,{\rm K}$ in steps of 200 K
- Gravity: same as MARCS 2005
- Metallicity: $-3.0 \leq [{\rm Fe/H}] \leq -1.50$ dex in steps of 0.5 dex
- Alpha: Poor, $[\alpha/{\rm Fe}] = 0.00$ for all models.

The Solar Spectrum



Source: Kurucz